Maple Conference 2020

November 2 - 6, 2020 **VIRTUAL**

Event Guide

Table of Contents

Welcome to the 2020 Maple Conference!

This conference is dedicated to exploring different aspects of the math software Maple, including Maple's impact on education, new symbolic computation algorithms and techniques, and the wide range of Maple applications. Attendees will have the opportunity to learn about the latest research, share experiences, and interact with Maple developers.

Where to Go

This year's Maple Conference is presented online in an **interactive virtual environment** complete with a presentation theater, networking lounge and resource library.

- To watch the recorded presentations, attend the live keynote presentations and discussion panels, and join the outing to the art gallery, go to the **THEATER**.
- To attend a live Q&A Session or participate in a Virtual Table, please go to the **LOUNGE**.
- In the **RESOURCE LIBRARY,** access a variety of written and visual materials.
- Go to **PROFILE** on arrival to fill in your virtual name tag, and check back later to see what badges you've earned in the Maple Badge Game.
- For support questions related to the virtual event environment, click on **HELP**.
- To ask questions about the conference or anything related to Maplesoft, go to the **LOBBY** and visit the Maplesoft Help Desk.

Overview Schedule

All times are given in Eastern Standard Time (EST).

	Sun Nov. 1	Mon Nov. 2	Tue Nov. 3	Wed Nov. 4	Thur Nov. 5	Fri Nov. 6
8am-9am		Virtual Tables	Virtual Tables	Virtual Tables	Virtual Tables	Virtual Tables
9am-10am	**Tutorials:** Introduction to Maple Programming AND Object Oriented Programming in Maple	**Keynote** Dr. Gabor Domokos	**Keynote** Dr. Laurent Bernardin	**Tom Thomson Art Gallery** Virtual Tour	**Meet the Developers** Panel Discussion	**Keynote** Dr. Juana Sendra Pons
10am-10:10am		Break	Break	Break	Break	Break
10:10am-11am		Q&A - Maple in Education	Q&A - Applications of Maple	Q&A - Maple in Education	Q&A - Applications of Maple	Q&A - Maple in Education
11am-11:10am		Break	Break	Break	Break	Break
11:10am-12pm		Q&A - Algorithms and Software	Q&A - Maple in Mathematics	Q&A - Algorithms and Software	Q&A - Maple in Mathematics	Q&A - Algorithms and Software
12pm-1pm		Virtual Tables	Virtual Tables AND Maple Ambassador Meeting	Virtual Tables	**Lessons From Social Media** Panel Discussion	
1pm-4pm				**Tutorial:** Building Interactive Applications in Maple		

Detailed schedules for the Q&A sessions and virtual tables can be found in the following pages.

Anti-Harassment Policy

The Maple Conference is dedicated to providing a harassment-free conference experience for everyone. We do not tolerate harassment of conference participants in any form. Conference participants violating these rules may be expelled from the conference at the discretion of the conference organizers. Participants asked to stop any harassing behavior are expected to comply immediately. You can report a violation of this policy to any Maplesoft staff or by emailing **mapleconference@maplesoft.com.**

Territorial Acknowledgment

Maplesoft's headquarters is located in Waterloo, Ontario, Canada. We acknowledge that Waterloo Region, including the three cities and four townships, is located on the traditional territory of the Haudenosaunee, Anishnaabe and Neutral People. We acknowledge the enduring presence of the Indigenous people with whom we share this land, their achievements and their contributions to our community. We offer this acknowledgement as an act of reconciliation between Indigenous and non-Indigenous peoples of Canada.

LIVE Q&A Sessions at the 2020 Maple Conference

Do you have questions for the presenters of the recorded On Demand presentations? Make sure you attend the **live Q&A sessions in the Lounge!**

Q&A Schedule

Monday Nov. 2, 10:10-11:00	**Maple in Education** | Moderated by Eithne Murray
Design, Construction and Use of Video Instructional Series based on "Maximum Productivity – Minimal Coding" Paradigm | *Scot Gould*	
Applications of DifferentialGeometry in Multivariable Calculus | *Frank Wang*	
CAS assisted study of envelopes: families of surfaces | *Thierry Dana-Picard*	
A simplified introduction to virus propagation using Maple's Turtle Graphics package | *Eugenio Roanes-Lozano*, *Carmen Solano-Macias* and *Eugenio Roanes-Macias*	
Rational Trigonometry using Maple | *Thomas Schramm*	
Pure Maths problem solved by Maple Maths Power | *Siyuan Deng* and *David J. Jeffrey*	
Using Maple for Studying Charge Transport in Nanoscale Devices with Undergraduates | *Erik Hoy*	

Monday Nov. 2, 11:10-12:00	**Algorithms and Software** | Moderated by John May
Machine Learning to Select Variable Orderings for Maple Algorithms | *Matthew England*	
Merging Maple and GeoGebra Automated Reasoning Tools | *Tomas Recio*, *Zoltan Kovacs* and *M. Pilar Velez*	
Rust for developing Fast, Parallelised Computer Algebra Systems | *Will Cashman*	
Algorithmic methods for filiform Lie algebras | *Francisco-Jesus Castro-Jimenez* and *Manuel Ceballos*	
Certified evaluations of Holder continuous functions at roots of polynomials | *Parker Edwards*, *Jonathan Hauenstein* and *Clifford Smyth*	
Machine Learning in Maple | *Stephen Forrest*	
Power series representations of hypergeometric type functions | *Bertrand Teguia Tabuguia* and *Wolfram Koepf*	
Decoupling Multivariate Fractions | *François Lemaire* and *Adrien Poteaux*	

Q&A Schedule

Tuesday Nov. 3, 10:10-11:00	**Applications of Maple** | Moderated by Orang Vahid

A Maple Toolchain for Rigid Body Dynamics of Serial, Hybrid and Parallel Robots | *Moritz Schappler* and *Tobias Ortmaier*

The Trials, Tribulations, and eventual Triumph of Computing and Optimising Fisher Information for Partially Observable Simple Birth Processes with Maple | *Matthew Skerritt*

Operations on nucleotide sequences to get some parameters of evolution | *Ernesto Alvarez*

Interpretations of Solar Waves and Oscillations with Maple | *Ram Ajor Maurya*

Analysis of semiclassical solutions of the quantum momentum rate equations | *Melvin Brown*

Maple Whiteboard - tactile, responsive calculations for science, engineering and technical analysis | *Samir Khan*

Branching out into structural identifiability analysis with Maple | *Jason Whyte*

Tuesday Nov. 3, 11:10-12:00	**Maple in Mathematics** | Moderated by Valerie McKay-Crites

Using Maple to study the eigenvalues of Q-matrices and P-matrices | *Laureano Gonzalez-Vega*

Spectral Problem for a Triple Differentiation Operator with Asymmetric Weight | *Serik Jumabayev* and *Daulet Nurakhmetov*

Using Maple to Approach Lüroth's Problem | *Jorge Caravantes*, *Sonia Pérez-Díaz* and *J. Rafael Sendra*

C-space Analysis using Tropical Geometry | *Abhilash Nayak*

Application of the Identify Command to Special Functions | *Thomas Richard*

Use of Maple to investigate length-preserving directions | *Juan Tolosa*

Algebraic aspects of a rank factorization problem arising in vibration analysis | *Elisa Hubert*, *Axel Barrau*, *Yacine Bouzidi*, *Roudy Dagher* and *Alban Quadrat*

Wednesday Nov. 4, 10:10-11:00	**Maple in Education** | Moderated by Paulina Chin

Development of Problem Solving Skills with Maple in Higher Education | *Cecilia Fissore*, *Marina Marchisio*, *Fabio Roman* and *Matteo Sacchet*

Student satisfaction determinants in hybrid learning environments based on MAPLE | *Tilo Wendler*, *Lisa Fischer* and *Marcel Dux*

Research and Practice of College Mathematics Curriculum Innovation Project based on Maple & CT (Computational Thinking) | *Guoxing Dai*, *Rongmei Dai* and *Dianc Lu*

Using Maple in Humanities-Related Assessment: Exploring Beyond Mathematical Boundaries in Education | Ananthan Ambikairajah and *Christopher Tisdell*

Maple for Distance Education in Secondary Schools during the Emergency from Covid-19 | *Cecilia Fissore*, *Francesco Floris*, *Marina Marchisio* and *Matteo Sacchet*

Students working SMARTER | *Alexander Rusnak*

Introducing Maple Calculator and Maple Learn | *Karishma Punwani*

Q&A Schedule

Wednesday Nov. 4, 11:10-12:00	**Algorithms and Software** | Moderated by Jürgen Gerhard

Rule-based Integration Using {Maple} | *David J. Jeffrey*, *Mahsa Kazeminooreddinvand* and *John May*

AGADE---a Maple package for computing rational general solutions of algebraic ODEs | *Johann Josef Mitteramskogler*

The TruncatedSeries Package for Solving Linear Ordinary Differential Equations Having Truncated Series Coefficients | *Sergei Abramov*, *Denis Khmelnov* and *Anna Ryabenko*

Puiseux Series Solutions of Autonomous AODEs | *Francois Boulier*, *José Cano*, *Sebastian Falkensteiner* and *Rafael Sendra*

Blends in Maple | *Robert Corless* and *Erik Postma*

Our Favorite Things: Maple 2020 Gems You May Have Missed | *Samir Khan* and *Karishma Punwani*

Multivariate Power Series in Maple | *Mohammadali Asadi*, *Alexander Brandt*, *Mahsa Kazemi*, *Marc Moreno Maza* and *Erik Postma*

Taylor Series Solutions of Delay Differential Equations Using dsolve solver in Maple | *Samir Hamdi*

Thursday Nov. 5, 10:10-11:00	**Applications of Maple** | Moderated by Nadia Sid

Modelling and sensitivity analysis of nonlinear firefighting systems using Maple | *Flóra Hajdu*, *Győző Molnárka* and *Rajmund Kuti*

Using Maple to solve real-time scheduling problems | *Jean-François Hermant*

Numerical solution for radial distortion rectification in optical systems | *Obed Isai Rios-Orellana*, *Rigoberto Juarez-Salazar* and *Victor Hugo Diaz-Ramirez*

Generate Captivating Visualizations with Maple | *Valerie McKay-Crites*

Maple as a Tool in HTCondor-managed High-Throughput Computing Environments | *Rémi Stellian*

Estimation of Travel Times for Additional Metrobus Route | *Verónica Nieves* and *Patricia E. Balderas*

Thursday Nov. 5, 11:10-12:00	**Maple in Mathematics** | Moderated by Thomas Richard

Conway Type Generalizations of The 3x+1 Problem and p-adic Representation of Rational Numbers | *Yagub Aliyev*

An Isometric Embedding of the Impossible Triangle into the Euclidean Space of Lowest Dimension | *Zhenbing Zeng*, *Yaochen Xu*, *Zhengfeng Yang* and *Zhi-Bin Li*

Computation of the L_∞-norm of finite-dimensional linear systems | *Yacine Bouzidi*, *Alban Quadrat*, *Fabrice Rouillier* and *Grace Younes*

Analytic Approximation for the Dirichlet Problem | *Robert Lopez*

Local analysis of separatrices around saddle points in autonomous two-dimensional dynamic systems | *Luis Sainz de Los Terreros* and *Antonio Rodriguez Mesas*

Using Maple to deal with offsets to quadrics | *Laureano Gonzalez-Vega*, *Jorge Caravantes*, *Mario Fioravanti* and *Gema M. Diaz-Toca*

A Maple Solution to the Problem 6 of the IMO 1988 | *Zhenbing Zeng*, *Yong Huang*, *Yaochen Xu*, *Xiaoru Chen* and *Lu Yang*

Q&A Schedule

Friday Nov. 6, 10:10-11:00	**Maple in Education** | Moderated by Matt Calder
Using Maple to Teach the SIR Model | *Douglas B. Meade*	
Quantum Chemistry Toolbox in Maple for Research and Education | *Jason Montgomery* and *David Mazziotti*	
An Early Introduction to the Frenet-Serret Frames in Calculus III: A project-based approach using Maple | *Lancelot Gooden*	
Understanding math concepts in music and vice-versa | *Gabriel Picioroaga*	
The Use of Maplesoft in an Honors Calculus Course | *Matthew Westerhoff*	
Maple at TAMU SEE-Math | *Philip Yasskin*	
Maple Interactive Texts and Simulations for Infectious Disease Modeling Projects | *John Pais*	

Friday Nov. 6, 11:10-12:00	**Algorithms and Software** | Moderated by Samir Khan
Sparse polynomial interpolation and computing roots of polynomials over prime fields | *Michael Monagan*	
The Inverse Gamma Function and its Numerical Evaluation | *Ana Carolina Camargos Couto*, *David Jeffrey* and *Robert Corless*	
A Maple implementation of the finite element method for solving boundary problems of the systems of ordinary second order differential equations | *Alexander Gusev*, *Sergue Vinitsky*, *Vladimir Gerdt*, *Ochbadrakh Chuluunbaatar*, *Galmandakh Chuluunbaatar*, *Le Hai Luong* and *Eugene Zima*	
Quantifier Elimination and projection & lifting Cylindrical Algebraic Decomposi-tions in the QuantifierElimination Package in Maple | *Zak Tonks*	
Factoring a Multivariate Polynomial Represented by a Black Box - A Maple to C Implementation | *Tian Chen* and *Michael Monagan*	
Detecting Invertible Linearizability Using the MapDETools Library | *Zahra Mohammadi* and *Gregory Reid*	
Simplification in Maple: Lambert W | *David J. Jeffrey*, *Toufic Ayoub* and *Kishore Basu*	
A Maple implementation of FFT-based algorithms for polynomial multipoint evaluation, interpolation, and solving transposed Vandermonde systems | *Kimberly Connolly*	

Virtual Tables at the 2020 Maple Conference

... in the Lounge

Join a **Virtual Table** in the Lounge during breakfast and lunch for informal conversations with other attendees. Each table has a broadly defined theme to help like-minded people find each other. You can discuss topics related to the day's conference streams, explore in a variety of special topics, or drop by a social table for general conversation.

Special Topics

Maple Technical Questions: Do you have a Maple question? Now's your chance to ask a Maplesoft expert!

Maple Programming Practices: Talk to Maple programming experts, ask questions, and share your Maple programming experiences and best practices.

Teaching Remotely: Share your experiences about how things are going, what's working and not working for you, pick-up tips from others, and if needed, gently vent to emphatic listeners.

Engaging Students: Looking for suggestions on engaging math students? Have some ideas or experiences to share? Come join other educators to talk about ways to engage math students.

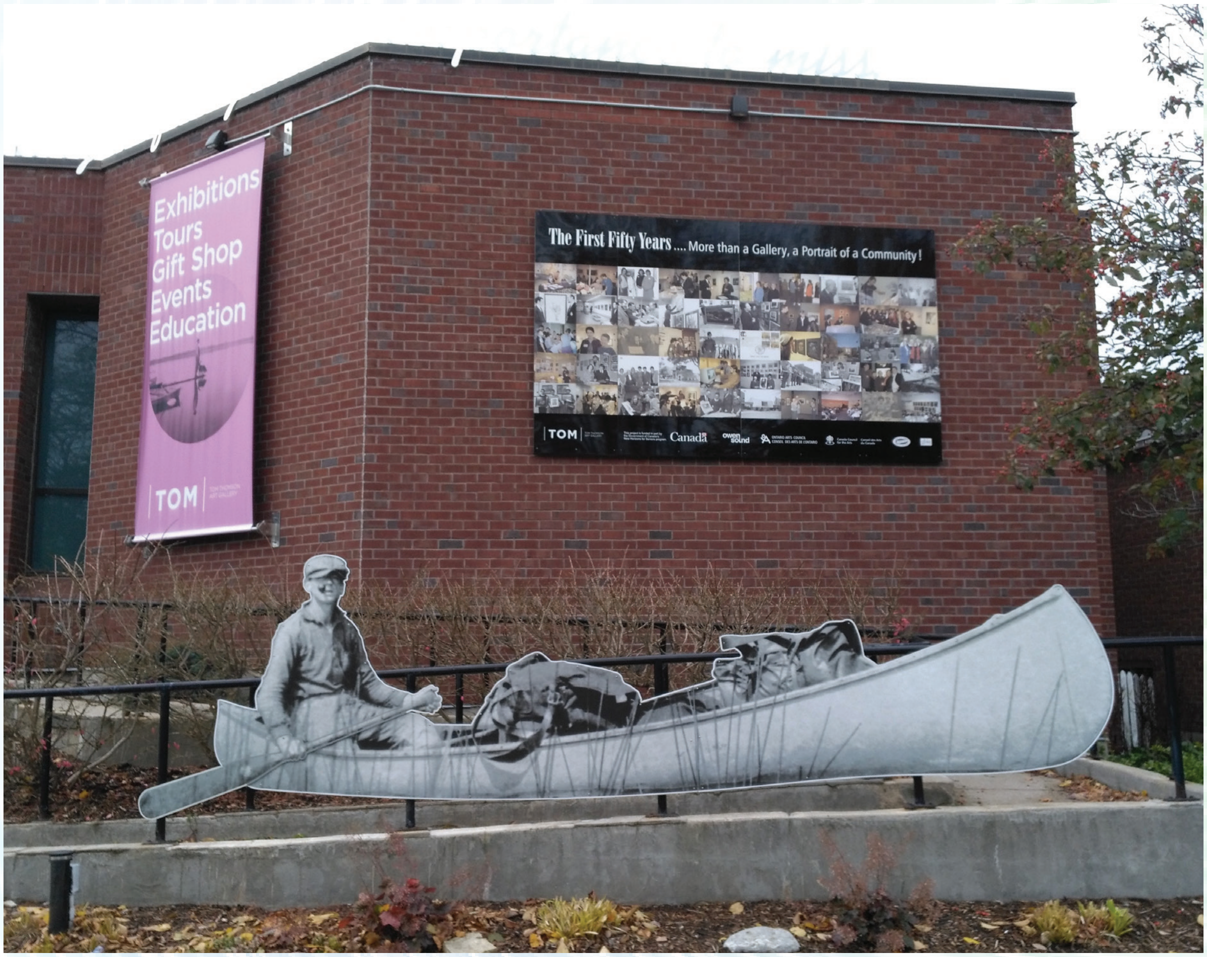
Maple Calculator and Maple Learn: Ask questions, request features, and share ideas and experiences about how these products do or could enhance your courses.

Table Schedule

Monday Nov. 2	Tuesday Nov. 3	Wednesday Nov. 4	Thursday Nov. 5	Friday Nov. 6
		Breakfast Tables - 8:00 - 9:00		
Maple in Education with Bruce Char, Drexel University	**Applications of Maple** with Laureano Gonzalez Vega, CUNEF	**Maple in Education** with Bruce Char, Drexel University	**Applications of Maple** with Laureano Gonzalez Vega, CUNEF	**Maple in Education** with Doug Meade, University of South Carolina
Algorithms and Software with George Labahn, University of Waterloo	**Teaching Remotely** with Lance Gooden, Johnston Community College, and with M. Pilar Vélez Melón, Universidad Nebrija	**Algorithms and Software** with Manuel Kauers, Johannes Keppler University	**Maple Technical Questions** with Erik Postma and Valerie McKay-Crites, Maplesoft	**Algorithms and Software** with Manuel Kauers, Johannes Keppler University
Social Table with Laurent Bernardin, Maplesoft	**Maple in Mathematics** with Thomas Richards, Maplesoft	**Maple Programming Practices** with Jürgen Gerhard and Erik Postma, Maplesoft	**Maple in Mathematics** with Thomas Richards, Maplesoft	**Maple Calculator and Maple Learn** with Laurent Bernardin and Karishma Punwani, Maplesoft
Social Table Rob Corless, Western University	**Social Table** with Stephen Watt, University of Waterloo	**Social Table** with George Labahn, University of Waterloo	**Social Table** with Jürgen Gerhard, Maplesoft	**Social Table** with Veronika Pillwein, Johannes Keppler University
		Lunch Tables - 12:00 - 1:00		
Maple in Education with Doug Meade, University of South Carolina	**Applications of Maple** with Wen-shin Lee, University of Antwerp	**Maple in Education** with Lance Gooden, Johnston Community College		
Algorithms and Software with Michael Monagan, Simon Fraser University	**Maple Programming Practices** with Jürgen Gerhard and Erik Postma, Maplesoft	**Algorithms and Software** with Michael Monagan, Simon Fraser University		
Maple Technical Questions with Erik Postma and Valerie McKay-Crites, Maplesoft	**Maple in Mathematics** with David Jeffrey, Western University	**Engaging Students** with Robert Lopez, Maplesoft and with Scot Gould, W.M. Keck Science Center The Claremont Colleges		
Social Table with David Bailey, University of California - Davis	**Social Table** with Ilias Kotsireas, Wilfrid Laurier University	**Social Table** with David Jeffrey, Western University		

Maple Conference
November 2 - 6, 2020

Tour of the Tom Thomson Art Gallery



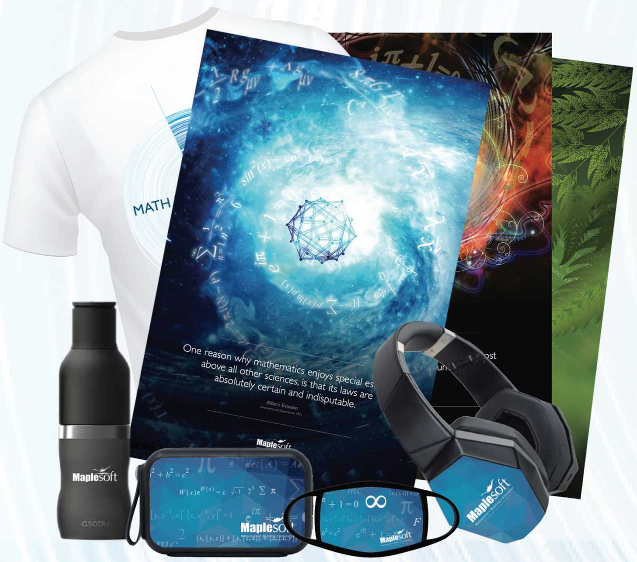
Please join us for a live and exclusive tour of the Tom Thomson Art Gallery's "Group of Seven: View from Here" exhibit on **Wednesday, Nov. 4 at 9am EST**. Access the Zoom link from inside the **THEATER**.

In this exhibit, the Tom Thomson Art Gallery celebrates the 100th anniversary of the formation of the Group of Seven. An artistic alliance, the members of the Group of Seven were bound by their desire to record the Canadian geography in a new and distinctive way. Showcasing the Gallery's impressive collection, "The View from Here" offers an intimate perspective on the artists that helped shape the Canadian nation.

Curated by David Huff, Curator of Collections at the Tom Thomson Art Gallery, with assistance from Joan Hawksbridge and Emily Dykeman.

Maple Badge Game

We all know that online conferences can't offer the same experience as an in-person event. We are working hard to make this event a good, interactive experience, but we recognize that this format also requires effort on your part. So, as a thank you, this year we are going to reward participation in a very tangible way – Maple swag!



Actual prizes may vary

All Maple Conference attendees will be automatically enrolled to participate in our Maple Badge Game. Badges are awarded for completing specific challenges and have an allotted point value. Once you complete a challenge, the virtual badge will be displayed in your profile and you will automatically earn that badge's point value.

Everyone with 50 points or more earns a reward. Posters! T-shirts! Fun masks! The more points you earn, the more rewards you will receive. Plus, the highest point earners will be entered into a draw for something extra special.

Following the close of the Maple Conference, we will send your reward(s) to the address you provided at the time of registration. **If you want us to send your reward to a different address, please update the address in your Profile before the end of the conference**.

Badge List

PROFILE MASTER
Complete the attendee profile, tour the virtual venue, and join the virtual conference during the live event week (Nov. 2- Nov. 6, 8am-1pm EST).
Earn 50 points

SESSION SEEKER
Attend three Q&A ses-sions.
Earn 150 points

LEADERSHIP
Attend all three keynote presentations.
Earn 150 points

SOCIALIZER
Join in three of the breakfast or lunch virtual table discus-sions. **Earn 150 points**

EXPLORER
Watch four recorded presentations from any one stream. **Earn 125 points**

INNOVATOR
Watch two recorded presentations from two different streams. **Earn 125 points**

DIGITAL PIONEER
Attend a Discussion Panel: Making Math Fun: Lessons from Social Media and/or Meet the Developers. **Earn 100 points**

GROUP OF SEVEN
Attend the virtual museum tour of the Tom Thomson Art Gallery. **Earn 100 points**

SOCIAL MEDIA STAR
Share your conference experiences on social media, using the social media buttons inside the virtual conference hall. **Earn 50 points**

MASTER MAPLE ENTHUSIAST
Watch eight recorded presentations and attend eight live Q&A sessions and one keynote or panel. **Earn 200 points**

Call for Papers

All presenters and invited speakers at the Maple Conference 2020 are invited to submit a full paper on the work they present at the conference. These papers will undergo peer-review, and if accepted, will appear in the conference proceedings as part of the Communications in Computer and Information Science series from Springer.

Paper submissions are due Dec. 31, 2020.

Visit the Maple Conference website for more details:
www.mapleconference.com.

Presentation Abstracts

Keynote Presentations

The Gömböc and the Shapes of Nature
Dr. Gabor Domokos

The Gömböc, whose original constructive proof relied on Maple computations, and the theory leading to its discovery is centrally connected to the understanding of the evolution of shapes in the non-living nature. Prof. Domokos will highlight the essence of this connection and illustrate it with examples from earth and planetary sciences.

Bohemian Matrices: Past, Present and Future
Dr. Juana Sendra Pons

A matrix family is called Bohemian if its entries come from a fixed finite discrete (and hence bounded) set, usually integers. We look at Bohemian matrices, specifically those with entries from {-1, 0, +1}. The name is a mnemonic for Bounded Height Matrix of Integers. Such families arise in many applications (e.g. compressed sensing) and the properties of matrices selected "at random" from such families are of practical and mathematical interest.

In this talk we present a Bohemian Matrices tour, exposing their appearance in the past, their promising present and their hopeful future.

Building the Right Tools for Math Education
Dr. Laurent Bernardin

Maplesoft has been building tools for math education for over 30 years. But building the "right" tools needs to take a lot of factors into account, from advancements in technology, to evolving trends in education, to the impact of a global pandemic. In this presentation, Dr. Laurent Bernardin, President and CEO of Maplesoft, will discuss Maplesoft's vision for how math tools should evolve to give educators and students the right tools for the times, for today, in the near feature, and beyond.

Discussion Panels

Meet the Developers
Want to know more about what goes on behind the scenes at Maplesoft? This is your opportunity ask questions of senior members of the Maplesoft R&D team. The panel will include people who are highly involved with the development of various aspects of Maple, the Maple Calculator app, and Maple Learn. Between them, this panel has many (!!) years of experience developing products for doing, learning, and teaching math. This is meant to be an interactive session, so come with lots of questions!

Laurent Bernardin, President and CEO

Andrew Smith, VP Product Development

Erik Postma, Manager – Mathematical Software Group

Paulina Chin, Software Architect

Jürgen Gerhard, Senior Director – Advanced Research

Paul DeMarco, Senior Director – Maple Development

Karishma Punwani, Director Product Management – Academic Market

Making Math Fun: Lessons from Social Media

While many educators have been thrust into the world of online learning very recently, there are some talented math enthusiasts with a vast amount of experience using their online presence to get students (and non-students!) excited about math. Join us for a discussion with prominent social media personalities in the math world, learn about some of the techniques they use to engage and excite students, and maybe even pick up some ideas you can use in your own teaching. This is meant to be an interactive session, so don't be shy about asking questions!

Tom Crawford, a.k.a TomRocksMaths, www.tomrocksmaths.com

Kyne Santos, a.k.a. Online Kyne, www.tiktok.com/@onlinekyne

Bobby Seagull, a.k.a. Bobby Seagull, www.bobbyseagull.com

Justice Shepard, a.k.a. JusticeTheTutor, www.tiktok.com/@justicethetutor

Please use **#mapleconference**
when sharing on social media!

Maple Workshops

Introduction to Maple Programming

Paulina Chin

In addition to being an interactive environment for problem-solving, visualization, and technical document preparation, Maple also features a powerful programming language that is especially useful for working with mathematics. Becoming familiar with the Maple language will allow to you increase the range and efficiency of what you can do in Maple, from writing short scripts to automate a repetitive calculation, to creating interactive applications for students and developing new algorithms to advance your research.

This introductory course is intended for those who already have some familiarity with Maple, but who have little to no experience with programming in Maple. This course will assume you have at least a small amount of experience with another programming or scripting language. People with no programming experience at all are still welcome to attend, but we recommend that they familiarize themselves with very basic programming concepts, such as loops and procedures, before the start of the course.

This course will cover the basics of the Maple language, common data structures, procedures and modules, tools to aid programming, and how to build simple interactive applications in the style of Maple's Math Apps.

Object Oriented Programming in Maple

Erik Postma

From the very beginning, it has been possible to program in Maple using some of the ideas of from object oriented programming, but not everyone is aware that modern Maple provides full support for objects and object-oriented programming. In this course, we will discuss object oriented programming in Maple, including:

- How objects work
- The underlying data structure of modules, and some of their features, such as ModuleApply and ModulePrint
- Static vs non-static members
- Overloading built-in commands
- Best practices

This course is intended for those who have some experience programming in Maple, and who want to learn how to use objects to write clean, modular, modern Maple programs.

Building interactive Applications in Maple

Nadia Sid

Maple documents can contain sliders, buttons, and other interactive elements that transform your document into an easy-to-use interactive application that requires no knowledge of Maple to use. Creating these applications in Maple is surprisingly easy, and does not require programming experience. In this course, you will learn:

- How to make use of the Explore facility to create applications that explore the parameters of any expression, using only a single function call or context menu operation
- How to build your own applications using Maple's interactive embedded components, including sliders, dials, gauges, math entry boxes, check boxes, and more
- Tips and tricks to ensure your application looks good and is easily maintainable
- Sharing your application through the MapleCloud, so your students and colleagues can access it easily, even if they don't have Maple

Attendees should have a basic familiarity with the Maple environment and using Maple commands.

Contributed Presentations

Contributed presentations are pre-recorded and can be watched at any time in the Theater. You can ask questions of the presenter during the live Q&A sessions in the Lounge.

Monday - Maple in Education

Design, Construction and Use of Video Instructional Series based on "Maximum Productivity – Minimal Coding" Paradigm.

Scot Gould (Claremont McKenna, Pitzer, Scripps).

Abstract. A collection of videos was created to help new users learn Maple using the maximum productivity – minimal coding (MPMC) paradigm for working with Maple. Target audience users were undergraduate in the physical sciences or engineering who would be expected to use Maple frequently each week in their courses, but would not be expected to solve all the problems using Maple.

The MPMC paradigm attempts to maximize the set of mathematics problems that can be solved using Maple, while generating readable mathematical documents, and minimizing, but not eliminating, the use of command-line commands. Like context-panel trained users, MPMC trained users employ clickable "input and manipulation of math" features of Maple, primarily those found in the palettes. However, MPMC trained users also learn command-line commands because commands often provide more flexibility and possess more options for the user than the current implementation of the context panel. MPMC documents use the 2d-input of Maple because more readers view the "traditional math display" of the 2-D input easier to comprehend than 1-D input. MPMC users work in the Worksheet mode because new users tend to find it easier to organize and understand the layout of their work. The paradigm attempts to serve the broadest range of users based on their efforts to use Maple.

The videos cover the most common mathematics required by undergraduate physical science and engineering students, i.e., through linear algebra and differential equations - ordinary and partial. Based upon recommendations from research in the field of educational, each video possesses the following attributes:

- a length of twelve minutes or less,
- the inclusion of intentional pauses during the presentation to make it easier for the viewer to stop the video and reproduce content before continuing,
- a limitation to the number of major concepts covered - usually three,
- integration of practices / example problems and solutions for the viewer to use to confirm their understanding of the content,
- a section on troubleshooting,
- the availability of downloading a Maple worksheet with additional practice problems and solutions.

This talk will cover the challenges and successes of 1) designing and implementing the MPMC paradigm, specifically, my attempt to minimize the number and complexity of Maple commands, 2) the implementation the attributes listed into each video, and 3) the experiences of, and assessment by, new users in an introductory undergraduate physics course.

Applications of DifferentialGeometry in Multivariable Calculus.

Frank Wang (LaGuardia Community College of the City University of New York).

Abstract. The calculus of differential forms is a powerful tool that unifies and simplifies many mathematical concepts. In this presentation we will demonstrate the utility of Maple's DifferentialGeometry package. We will perform double, triple, line and surface integrals in James Stewart's Calculus from a modern geometric point of view. Quoting Harley Flanders, a line integral leads us to the one-form, a surface integral leads us to the two-form, and a volume integral leads us to the three-form. We will use the Pullback command extensively to transform among coordinates and manifolds. The method of differential forms can inspire students to see the connection among these integrals. The Fundamental Theorem for Line Integrals, Green's Theorem, Stokes' Theorem, and Divergence Theorem can be elegantly written in a compact form which is valid in any dimension. Time permitting, we will consider two related topics: (1) use differential forms to derive key identities in thermodynamics; (2) prove that the integrating factor in the first-order linear differential equation and the integrating factor in exact differential equation are equivalent.

CAS assisted study of envelopes: families of surfaces.

Thierry Dana-Picard (Jerusalem College of Technology).

Abstract. We study envelopes of 1-parameter and 2-parameter families of surfaces in 3D space. The algebraic algorithms of Maple are used to solve the suitable systems of equations, then the graphical features (plots, animations) are used to analyze the output. Back to algebra, the conjectures about cusps and other properties may be proven. Such activities have been proposed to students as an extension of their curriculum.

A simplified introduction to virus propagation using Maple's Turtle Graphics package.

Eugenio Roanes-Lozano (Universidad Complutense de Madrid), **Carmen Solano-Macias** (Universidad de Extremadura) and **Eugenio Roanes-Macias** (Universidad Complutense de Madrid).

Abstract. In March 2020 the Spanish authorities ordered a nation wide home confinement in an effort to avoid the spread of COVID-19. The first two authors, parents of a teenager son, decided to prepare a very simplified justification (for kids), that was initially implemented in Scratch 3. It relates virus propagation to fractal trees and is based on the Turtle Geometry. It shows how one infected cat can spread a contagious illness in a cat colony (the average number of cats infected by each ill cat can be easily changed). This justification was later written as a tale and recorded in a 5 minutes video (in Spanish) that are available from the Instituto de Matemática Interdisciplinar of the Universidad Complutense de Madrid web page [1] and were presented at ESCO 2020. The second author teaches at a school of communication and information sciences and improved and reorganized the tale and leaded the development of the video. Note that the first and third authors have experience in designing and developing applications for teaching and decision making in medicine (with CAS). Moreover, they developed in the 1990's a Maple implementation of the Turtle Geometry [2] (that was incorporated to the Maple Share Library). Now, an improved new version of the tale (in English), which underlying code is written in Maple 2020 and takes advantage of using exact arithmetic for storing the geometric coordinates of the points, is available from MaplePrimes [3]. This advantage is important when dealing with geometric designs that somehow concatenate geometric objects, as is the case of fractals (something already mentioned in [4]). The possibilities of Maple's Turtle Geometry package and this particular application will be analyzed.

Rational Trigonometry using Maple.

Thomas Schramm (HafenCity Universität Hamburg).

Abstract. In 2005, Norman Wildberger presented a concept for a geometry without transcendental functions in his book "Divine Proportions: Rational Geometry for Universal Geometry". Inspired by ancient Babylonian and Greek mathematics, he introduces spreads and quadrants instead of angles and lengths to describe triangles and more. With this concept, all tasks and proofs of Euclidean geometry can easily be carried out without sine and cosine functions and without introducing a differential calculus.

In a series of Maple worksheets, we introduce the concept and definitions and compare some basic calculations to the "normal way". This concept has a clear didactic advantage and shows some parallels to the way surveyors carry out their calculations, avoiding transcendental functions wherever they can.

Pure Maths problem solved by Maple Maths Power.

Siyuan Deng (University of Western Ontario) and **David J. Jeffrey** (University of Western Ontario).

Abstract. A problem in pure mathematics concerning completely monotonic functions is considered. Although problems of this type are usually approached using a variety of pure mathematical techniques, here we describe a computational approach. We use Maple's power to handle very large expressions in our computations.

The function we consider is $f(x;a,b)=e^{a} - (1 + a/x)^{x-b}$. This function will satisfy the requirements for being completely monotonic (CM) if it decays monotonically to zero on the interval $x \in (0,\infty)$.

Further, we require that all derivatives $d^{n} f/dx^{n}$ also decay monotonically to zero. We know that if $a>3$ then a,b is not CM.

We also know that if $a\leq 1$, then $f(x;a,0)$ is CM. Thus there are critical values of a,b which determine the transition from not-CM to CM. These critical values are what we calculate.

In order to obtain these critical values, our method requires us to consider derivatives up to the order of 100,000. No human can differentiate 100,000 times, but Maple can. The numerical calculations become extremely delicate and sensitive as the number of derivatives increases, but again Maple's computational power comes to the rescue. We can readily compute to 30, or 50, decimal digits and obtain very precise estimates of the critical parameters.

Using Maple for Studying Charge Transport in Nanoscale Devices with Undergraduates.

Erik Hoy (Rowan University).

Abstract. Since the 1970s, the question of how to construct nanoscale electronic devices such as switches, resisters, and transistors using only a single molecule has attracted significant experimental and theoretical interest. Compared to traditional electronic components, these devices offer both additional miniaturization potential and unique charge transport properties. This area of research is an exciting one for undergraduates offering them the ability to help design their own novel nanoscale device. Designing effective single molecule devices, however, requires an accurate theoretical description of charge transport. The current standard approach for studying single molecule charge

transport problems is the non-equilibrium Green's function formalism (NEGF). While this formalism is highly effective, using traditional NEGF software packages creates notable issues for undergraduate researchers. The software packages are typically complex and designed with graduate students in mind and as such lack an intuitive graphic user interface. Addressing these issues typically requires purchasing expensive, specialized packages that are difficult for undergraduate institutions to afford. Using Maple and its Quantum Chemistry Toolbox, we have developed a low-cost alternative in the form of a series of intuitive, undergraduate friendly Maple workbooks and scripts. This allows for quantum chemistry calculations from Maple to be integrated with external, freely available NEGF software. Maple's workbooks provide with an intuitive graphical interface allowing undergraduates to easily run charge transport calculations. This opens up a whole new areas of nanoscale research typically considered too difficult for undergraduates. In this presentation, we will discuss the results several different studies that undergraduates have already performed using Maple and how this work can be used to bring cutting edge research into the lab and classroom.

Monday - Algorithms and Software

Machine Learning to Select Variable Orderings for Maple Algorithms.

Matthew England (Coventry University).

Abstract. This talk will survey our already published work in CICM 2019, SC-Square 2019, MACIS 2019 and ICMS 2020 on the development of Machine Learning (ML) technology for selecting the variable ordering of a symbolic computation algorithms in Maple. The choices made by the ML classifiers do not effect the mathematical correctness of the end result, but can have a great impact on the resources required to find them. There is great potential to use ML to optimisation computer algebra systems in this way.

We will describe our experiments with different ML classifiers algorithms in sklearn, our new technique for feature extraction from polynomials, how we have adapted sklearn's cross validation procedure for hyper parameter selection to better suit this application, and how we have packaged the above into a freely available software pipeline.

Our experiments were all for selecting the variable ordering used by Cylindrical Algebraic Decomposition in the Maple Regular Chains Library, but the pipeline could be easily applied for any other procedure which takes polynomials as input and requires a variable ordering.

This was joint work with Dorian Florescu, funded by EPSRC Project EP/R019622/1: Embedding Machine Learning within Quantifier Elimination Procedures.

Merging Maple and GeoGebra Automated Reasoning Tools.

Tomas Recio (Universidad de Cantabria), **Zoltan Kovacs** (The Private University College of Education of the Diocese of Linz) and **M. Pilar Velez** (Universidad Antonio de Nebrija).

Abstract. A branch of the Automated Deduction in Geometry (ADG) theory deals with the automatic proof and discovery of theses holding on a given collection of hypotheses. The mechanical proof and derivation of such statements, through computational complex algebraic geometry methods, presents some challenging issues regarding the translation in algebraic terms of the given geometric facts, the verification or the finding of the sought properties, and the interpretation of the outcome.

The dynamic mathematics program GeoGebra already provides some extraordinary performing automated reasoning tools [2], although sometimes yielding quite surprising (for human intuition) outputs, mostly due to the difficulties of the translation geometry-algebra process.

In our presentation we will argue how some challenging issues rising in this context could be be better addressed through the collaboration of GeoGebra Automated Reasoning Tools and some Maple packages for polynomial ideal manipulation, through the concours of recently introduced theoretical concepts dealing with the notions of minimal extended polynomial [1] and true on parts [3]. Moreover, we will describe the potential role of some new approaches concerning specific protocols for testing zero dimensional hypotheses ideals and for the automatic discarding of intuitively degenerate components, avoiding the need to compute primary decompositions, by using Saturation and Gröbner basis computation over polynomial rings with rational function fields as field of coefficients.

Rust for developing Fast, Parallelised Computer Algebra Systems.

Will Cashman (The Australian National University).

Abstract. The goal of a computer algebra system is to provide a convenient platform for efficient computations with a diverse range of mathematical objects. When it comes to implementing such a system, there has always been a divide between fast programming languages like C and Fortran, and expressive languages like Lisp and Haskell.

Rust is a relatively new programming language which attempts to bridge this gap. It combines the mathematical expressiveness of Haskell's type-classes and algebraic data types, while keeping the low-level control of C; two qualities which make it perfect for developing large, complex computer algebra systems. Additionally, it offers a provably safe concurrency model for scalable parallelism to ensure computations are executed as fast as possible.

Already Rust has libraries which formulate the notion of both algebraic (group, ring...) and analytic (affine space, metric space ...) algebras. This provides developers with a common interface to write algorithms for general classes of mathematical objects in which the language checks the inputs are members of the correct algebras at compile time. This also paves the way for enhanced program verification/fuzzing tools for your algorithms.

In this talk, we will see how we can leverage Rust's generic programming system to create powerful zero-cost abstractions for effective development of computer algebra system. Namely, we will cover how mathematical algebras are formulated and used, and how we can safely parallelise the code to obtain the greatest performance. Throughout this talk, I will draw examples from my own publicly available Rust library for a generic polynomial type with efficient polynomial operations (in particular, polynomial multiplication) to illustrate key concepts.

Algorithmic methods for filiform Lie algebras.

Francisco-Jesus Castro-Jimenez (Universidad de Sevilla) and **Manuel Ceballos** (Universidad Loyola Andalucía).

Abstract. In this paper, we present three different algorithms, implemented in Maple, dealing with (complex) filiform Lie algebras. These Lie algebras were introduced by M. Vergne [6] in the late 1960s. Filiform Lie algebras have been described and classified up to dimension *8* (see E. Remm [4]).

Our first algorithm computes the law of a finite-dimensional filiform Lie algebra taking as input its dimension and the value of two numerical invariants that were introduced in Echarte-Núñez-Ramírez [3].

The derived length of a filiform Lie algebra has been treated in Burde-Dekimpe-Vercammen [1] and Castro-Jiménez-Ceballos-Núñez [2].

Our second algorithm computes the derived length of a finite-dimensional filiform Lie algebra starting from its dimension.

According to Th. 3.6.6 in Varadarajan V.S. [5], every finite-dimensional nilpotent Lie algebra is isomorphic to a subalgebra of Lie algebra g_m, of m x m strictly upper-triangular matrices, for some positive integer m. Our last algorithmic method computes the minimal m such that a given finite-dimensional filiform Lie algebra g is isomorphic to a subalgebra of g_m.

Certified evaluations of Holder continuous functions at roots of polynomials.

Parker Edwards (University of Notre Dame), **Jonathan Hauenstein** (University of Notre Dame) and **Clifford Smyth** (University of North Carolina at Greensboro).

Abstract. Certified estimates for roots of polynomials can be obtained using various methods such as the Kantorovich theorem and Smale's alpha-theory based on Newton's method. In addition to computing roots, many applications in science and engineering utilize the value of various functions evaluated at the roots. For example, critical values are the value of an objective function evaluated at critical points. When evaluation functions are analytic, a natural application of Newton's method yields certified estimates. However, these estimates no longer apply for Holder continuous functions which are a generalization of Lipschitz continuous functions where continuous derivatives need not exist. This work develops and analyzes an alternative iteration scheme which yields certified estimates of evaluating Holder continuous functions at roots of polynomials. An implementation of the method is developed in Maple to demonstrate its efficacy and efficiency. A variety of examples using this Maple code will also be presented.

Maplesoft Presentation: Machine Learning in Maple

Dr. Stephen Forrest (Maplesoft)

Abstract. The past few years have seen explosive growth in machine learning, particularly in artificial neural networks ("deep learning"). Learn how you can apply Maple's built-in machine learning tools to train and experiment with models and get an exclusive advance look at some of the new machine learning features we are working on.

Power series representations of hypergeometric type functions.

Bertrand Teguia Tabuguia (University of Kassel) and **Wolfram Koepf** (University of Kassel).

Abstract. In 1992, Koepf proposed an algorithmic approach for power series computations. This algorithm was extended for a larger family of expressions thanks to Petkovsek's and van Hoeij's algorithms (1993 and 1998) which compute hypergeometric term solutions of any given holonomic recurrence equation (RE). Mark van Hoeij's algorithm whose outputs are bases is available in Maple through the command "LREtools[hypergeomsols]", and Koepf's algorithm through "convert" and the built-in module "FormalPowerSeries".

However, using van Hoeij's algorithm one cannot compute m-fold hypergeometric term solutions of holonomic REs, for integers m. Given a field K of characteristic zero, a term a(n) is said to be m-fold hypergeometric if the ratio term a(n+m)/a(n) is rational over K. Note that the hypergeometric term case corresponds to m=1. If one adds for example an odd hypergeometric function and an even hypergeometric function (which both are two-fold hypergeometric), then van Hoeij's algorithm cannot find those by solving the resulting recurrence equation. Due to this limitation, the computation of many power series are missed by Maple, in particular, linear combinations of power series having m-fold hypergeometric term coefficients are generally not detected.

We overcome these issues by using a new algorithm called mfoldHyper, proposed in the Ph.D. thesis of the first author to compute bases of the subspace of m-fold hypergeometric term solutions of holonomic REs. It turns out that mfoldHyper linearizes the computation of hypergeometric type power series, i.e. every linear combination of hypergeometric type power series is detected.

Decoupling Multivariate Fractions.

François Lemaire (CRIStAL, University of Lille) and Adrien Poteaux (CRIStAL, University of Lille).

Abstract. We present an algorithm for decoupling multivariate fractions. The goal is to enhance the readability of fractions and to improve interval arithmetics, thanks to the decomposition of the fraction into sum, product and nested decoupled fractions.

Tuesday - Applications of Maple

A Maple Toolchain for Rigid Body Dynamics of Serial, Hybrid and Parallel Robots.

Moritz Schappler (Institut für Mechatronische Systeme, Leibniz Universität Hannover), **Mark Wielitzka** (Institut für Mechatronische Systeme, Leibniz Universität Hannover) and **Tobias Ortmaier** (Institut für Mechatronische Systeme, Leibniz Universität Hannover).

Abstract. We present a fully-automated Maple toolchain for generating rigid body dynamics in symbolic form.

The peculiarity lies in the framework of Bash scripts controlling the full workflow of the toolchain: the optimized Matlab code generated by Maple is automatically converted to function files with proper documentation and input assertions.

A unit test framework is also generated.

All functions for the robot are completely tested, rendering manual post-processing of the output unnecessary.

The Trials, Tribulations, and eventual Triumph of Computing and Optimising Fisher Information for Partially Observable Simple Birth Processes with Maple.

Matthew Skerritt (CARMA, University of Newcastle).

Abstract. We present a recent development that has greatly expanded our capability to compute Fisher Information for Partially Observable Simple Birth Processes. The new technique uses dynamic programming techniques to compute a (high-dimensional) recurrence relation derived from a generating function.

Maple proved to be indispensable for computing the formal sums of the generating function. The dynamic programming task—at the scale we wished to compute it—proved entirely too much for Maple. We were nonetheless able to make use of Maple's optimisation library with an external library we wrote using C++ to more efficiently compute the Fisher recurrence.

We present the problem, and discuss the particulars of the above: the successes, the challenges, and the solutions.

Operations on nucleotide sequences to get some parameters of evolution.

Ernesto Alvarez (UNIVERSIDAD COMPLUTENSE DE MADRID).

Abstract. Let "N" be a set of lineages. They are sequences of nucleotides. One associates to N a graph, "T", called a phylogenetic tree, that explains all its ancestral relations. Branch lengths can be defined as expected number of substitutions. All sequences in N can be aligned to compare their sites. After the alignment, one counts different character patterns (and also substitution patterns). "Hadamard Conjugation" is an equation that relates all branch lengths to all substitution patterns. "Hadamard Conjugation" keeps that information into two spectral matrices. Througouth this talk, I can explain what has to be done, and how, to obtain the one that keeps the information of all substitution patterns. The other matrix is left as a parameter matrix (the topology of T is an assumption; branch lengths of T are parameters).

There are some difficulties when dealing with nucleotide sequences. The first difficulty is that "Hadamard Conjugation" assumes equal length sequences, but in practice different sequences have different length. This leads to the problem of "filling the GAPS" so as to get equal length sequences. The criteria used to fill de GAPS reduces variation among the visible character patterns. When computing substitution patterns, one chooses one of the sequences in N as a reference. The selection of the reference lineage, computation of substitution patterns and their location into the spectral matrix face some difficulties. All these operations among lineages (sequences) are done automatically by the use of some libraries on maple. These libraries are good tools that I want to share to the community. I will also show some real examples of biological meaning.

Interpretations of Solar Waves and Oscillations with Maple.

Ram Ajor Maurya (National Institute of Technology Calicut).

Abstract. Solar observations have revealed that the magnetohydrodynamic (MHD) waves and oscillations are ubiquitous in the solar atmosphere, with a wide range of periods. Their significance has increased due to their potential as a remote diagnostic tool to investigate the interior and atmospheric properties. There are two types of waves in the solar atmosphere, standing waves and propagating waves. The surface of Sun oscillates in million of patterns or modes with average period of five minutes. These oscillations are caused by the superposition of acoustic waves trapped below the photosphere. These modes are used to investigate the interior solar properties. When these waves interact with the magnetic field lines, they transfer their energy into the solar atmosphere. The running penumbral waves and oscillations are examples of propagating waves and can be used to explore the chromospheric dynamics. Propagating waves play a significant role in heating the solar atmosphere and acceleration of the solar wind. In this paper, we will present the theoretical interpretations and numerical simulations of waves and oscillations, as mentioned above, using Maple software.

Analysis of semiclassical solutions of the quantum momentum rate equations.

Melvin Brown (Birkbeck College University of London, UK).

Abstract. We analyse semiclassical solutions of the coupled real and imaginary momentum rate equations, as derived from the Schrodinger equation [1].

The semiclassical domain is spanned by the unitless parameter $0 < h < 1$ on the scale of Planck's unit of action ħ; thus $h = 0$ corresponds to classical physics, and $h = 1$ to quantum physics.
The momentum rate equations are formulated as a pair of coupled PDEs and are solved using MAPLE's solver pdsolve, with integration scheme parameter $0 < \Theta v < 1$. The solution space is thus parameterised by (h, Θv).

For each parameter set (h, Θv), solutions for the coupled real and imaginary momenta (pR(x,t),pI(x,t)) are respectively written to files and coplotted as animated graphs displaying the coupled dynamics of their solutions.

Associated with each solution file (h, Θv) is its filesize and its entropy (compressed filesize / uncompressed file size), thus displaying dependency upon h for given Θv . For example, transitions in entropy or filesize are graphically displayed on the domain of $0 < h < 1$.

The MAPLE worksheet contains many plots illustrating the dependency of the momentum solutions upon both (h, Θv). In particular, the worksheet illustrates methods for exploring the solution space, and of identifying and navigating the domain of the feasible solutions of the momentum rate PDEs in the (h, Θv) space.

Reference

[1] M. R Brown, "The symplectic and metaplectic groups in quantum mechanics and the Bohm interpretation", PhD Thesis, Birkbeck College, University of London, 2004.

Maplesoft Presentation: Maple Whiteboard - tactile, responsive calculations for science, engineering and technical analysis

Samir Khan (Maplesoft)

Abstract. Maple Whiteboard is a new calculation tool that lets you fluidly assemble your mathematical ideas and interactively place them into a formal technical structure while you work.

The interface replicates the design metaphor of a whiteboard – you can place math and text anywhere onscreen. Space and position are a core part of the computational model – simply move math to change the evaluation order.

Moreover, mathematical changes cascade immediately through the Whiteboard so that results are always up to date.

This talk will introduce Maple Whiteboard, demonstrate its major features and present several use cases.

Branching out into structural identifiability analysis with Maple.

Jason Whyte (CEBRA, School of BioSciences and ACEMS, School of Mathematics and Statistics, University of Melbourne).

Abstract. Suppose that we represent some physical system by a (model) structure — a collection of parametric relationships (e.g. differential equations), and constraints on the parameters. Prior to using the structure in predicting system behaviour, we must estimate the true parameter vector θ^* from system observations.

Parameter estimation may return multiple (even infinitely-many) equally valid estimates of θ^*. Inconveniently, distinct estimates may lead to very different predictions for unobservable state variables, or for outputs beyond the range of our data. If we cannot distinguish between alternative estimates, we cannot confidently use our structure for prediction. Consequently, our data collection and modeling efforts are unproductive.

Non-unique parameter estimates may follow inexorably from the combination of the study design (which includes planned inputs or controls), and the assumed structure. If so, we can anticipate this problem by testing the structure (with the planned inputs) for the property of structural global identifiability (SGI). Such a test does not require data. Instead, we assume an idealised situation where an infinite, error-free data record is available, and our structure correctly represents the system. Solving algebraic equations derived from the structure will show whether a unique estimate of θ^* is possible (but not guaranteed) under these idealised conditions. We do not expect a better result for real, noisy data. Further, test results may guide the reparameterisation of our structure so that parameter estimation will achieve a more favourable result.

In various domains, the testing of structures for SGI remains uncommon. This may reflect the specialised nature of identifiability analysis, which requires different skills from those employed in mathematical modeling. We may introduce some key features of identifiability analysis via study of "linear time-invariant" (LTI) structures. Such structures include a collection of linear, constant-coefficient ordinary differential equations that describe the time evolution of state variables. Testing LTI structures for SGI naturally motivates the study of various topics, including elements of systems theory, Laplace transforms, and systems of algebraic equations. Further, an understanding of LTI structures is relevant to active research areas, such as testing linear switching structures (which are piecewise LTI) for SGI.

We present Maple routines that enable the testing of LTI structures for SGI. Further, our code allows the user to experiment with how changes in the applied inputs or parameterisation may change the result of a test. The routines will assist the checking of results from alternative methods, or other symbolic algebra packages.

Tuesday - Maple in Mathematics

Using Maple to study the eigenvalues of Q-matrices and P-matrices.

Laureano Gonzalez-Vega (CUNEF Universidad).

Abstract. *P*-matrices are matrices all of whose principal minors are positive. *Q*-matrices are matrices whose sums of principal minors of the same order are all positive. A matrix is said to be a *PM*-matrix (resp. *QM*-matrix) if all its powers are *P*-matrices (resp. *Q*-matrices). We will show how Maple has been specially useful to deal with two issues. The first one is devoted to fully characterise the real *QM*-matrices A, $n \times n$, for n equal to $2, 3, 4$ and 5. And the second one is devoted to characterise those real matrices A, 4×4, such that A and A^2 are *Q*-matrices but not all eigenvalues of A have positive real parts.

Spectral Problem for a Triple Differentiation Operator with Asymmetric Weight.

Serik Jumabayev (Academy of Public Administration under the President of the Republic of Kazakhstan) and **Daulet Nurakhmetov** (Institute of mathematics and mathematical modelling).

Abstract. We study the spectrum of a class of two-point boundary value problems for an ordinary differential operator of triple differentiation with weight. We show that the spectrum of a problem with asymmetric weight and with periodic boundary conditions fills the entire complex plane. We present an example of a problem with asymmetric weight to which one cannot assign a given spectrum by changing only one of the boundary conditions. The real zeros of the characteristic determinant were computed with the use of the Maple software package.

We believe that our results can be useful for software developers of symbolic and symbolic-numeric methods.

Jumabayev, S.A., Nurakhmetov, D.B. Spectral problem for a triple differentiation operator with asymmetric weight. Diff Equat 53, 704–707 (2017). https://link.springer.com/article/10.1134/S0012266117050147#citeas

Using Maple to Approach Lüroth's Problem.

Jorge Caravantes (University of Alcalá), **Sonia Pérez-Díaz** (University of Alcalá) and **J. Rafael Sendra** (University of Alcalá).

Abstract. The geometric version of Lüroth's theorem states that for every, non necessarily birational, parametrization f(*t*) of an algebraic curve there exists a birational parametrization g(*t*) of the same curve as well as a rational function *R*(*t*) such that f(*t*)=g(*R*(*t*)). Moreover, there exist algorithmic answers to the problem (see [4]). For the case of rational surfaces, Castelnuovo's Theorem ensures that the answer is also true but under the hypothesis that the field is algebraically closed. However, from the computational point of view, the existing answers are not fully satisfactory. One possibility may consists in using the parametrization to compute the implicit equation of the surface, and afterwards apply a birational parametrization algorithm (see [3]). However, this solution may be unsatisfactory from the complexity point view. If we focus on approaches that stay working with parametric representations, one can apply the results in [1] and [2] where the problem is partially solved by reducing the question to the case of curves over a transcendental field extension of the ground field. However, the answer in the above papers does no cover all the cases, and the reparametrization problem is not totally solved and alternative approaches are needed.

Our idea is extending the results in [1] and [2] using generic fibers of the input parametrization. In particular, if $P(t)$ is the input parametrization, the key idea is to use the following property: there exists a birational parametrization Q satisfying that $P(t) = Q(R(t))$ if and only if $F_P(P(s)) = F_R(R(s))$, for some $R(t) \in (K(t) \cap K)^2$, where F_P represents the fiber of the map induced by the parametrization P; similarly F_R.

The derived method is not optimal in its current version and some additional ideas and further work is necessary to improve it. However, in some cases on which the fibre of the input parametrization can be explicitly computed the method provides an optimal solution to the reparameterization problem.

In this talk, besides the mathematical ideas, we plan to show a collection of Maple procedures on the topic and to illustrate the performance of the method with examples developed in Maple.

C-space Analysis using Tropical Geometry.

Abhilash Nayak (Laboratoire des Sciences du Numérique de Nantes (LS2N)).

Abstract. It is evidently crucial to identify and scrutinize the singular points of configuration spaces (C-space) of mechanisms. They usually appear when the manifold intersects itself, leading to different branches of motion. There exist many approaches to detect those intersections if they are transversal. However, the problem remains challenging if there are tangential, cuspidal, inter-dimensional or a combination of these intersections. This paper proposes an approach derived from tropical geometry to analyze the neighborhood of any point on the C-space of 1-dof mechanisms. This is done by finding the approximate rational parametrization of the curve(s) passing through the given point using Puiseux series. The proposed approach is shown to succesfully detect the transversal branchings in two foldable four bar mechanisms and a cusp in the configuration curve of the double Watt mechanism.

Maplesoft Presentation: Application of the Identify Command to Special Functions

Thomas Richard (Maplesoft)

Abstract. Maple's identify command is one of its hidden gems: unknown to many users, but a very powerful tool for obtaining exact values of special functions at given points, as well as definite integrals and sums of such functions. It is based on well-established methods like PSLQ (Partial Sum of Least Squares, by Bailey and Ferguson) and LLL (finding "short" vectors in lattices, by Lenstra, Lenstra and Lovász), both of which are implemented in the IntegerRelations package. Numerical evaluation followed by careful application of the identify command and its various options allows us to either reconstruct or to conjecture exact representations.

We are presenting examples from three function families:

1) algebraic values of EllipticModulus evaluated at exp(-s*Pi) where s is algebraic

2) semi-infinite integrals of quotients of Airy wave functions

3) deriving a finite summation formula for certain infinite series of Zeta(k)*k^m

While none of these observations are trivial, they differ in computational complexity. Particularly for the first example set, numerical evaluation at very high settings of Digits is required.

It is essential to emphasize that correctness of these representations remains to be proven - that's why they are not

Use of Maple to investigate length-preserving directions.

Juan Tolosa (stockton university).

Abstract. In 2015 Dr. Robert Lopez gave a presentation on eigenvalues and eigenvectors of matrices. As an aside, he raised the question of directions along which the norm of vectors are preserved under a given matrix. For a given integer matrix, it is interesting to find length-preserving directions determined by vectors with integer coordinages.

In this presentation I will illustrate how Maple helped in my research of this question for both 2x2 and 3x3 matrices.

Algebraic aspects of a rank factorization problem arising in vibration analysis.

Elisa Hubert (University of Lyon, LASPI), **Axel Barrau** (Groupe Safran), Yacine Bouzidi (Inria Lille), **Roudy Dagher** (Inria Lille) and **Alban Quadrat** (Inria Paris).

Abstract. This paper continues the study of a factorization problem arising in gear fault surveillance. The structure of a class of solutions - interesting in practice - of this factorization problem is studied. We show that these solutions can be parametrized. The parameter space P is proved to be the complementary of an algebraic set that is explicitly characterized based on module theory and computer algebra. A finite open cover of P is obtained and for each basic open subset of the cover, a closed-form solution is computed using computer algebra. Hence, the local structure of the solution space can be finely studied. Finally, we show that the existence of a single closed-form solution defined on the whole parameter space P is related to difficult problems in module theory.

Wednesday - Maple in Education

Development of Problem Solving Skills with Maple in Higher Education.

Cecilia Fissore (Università degli Studi di Torino), **Marina Marchisio** (Università degli Studi di Torino), **Fabio Roman** (Università degli Studi di Torino) and **Matteo Sacchet** (Università degli Studi di Torino).

Abstract. Problem solving is the ability to understand the environment, identify complex problems, and review related information to develop, evaluate strategies and implement solutions to build the desired outcome. It is the basis for creative thinking, new inventions, evolution, continuous improvement, communication, and learning. It is listed in the 21st century set of skills for students and future professionals.

Mathematics is the queen of sciences that boosts problem solving skills. In Higher Education all scientific degree programs there is at least a module in Mathematics that should develop students' problem solving skills beyond notions.

Mathematics Modules of the Biotechnology Bachelor Degree and of the Strategic Science Bachelor and Master Degrees at the University of Torino use innovative digital technologies, like the Advanced Computing Environment (ACE) Maple, and methodologies in order to facilitate the learning of Mathematics and the development of problem solving skills. Students at the beginning of the courses are required to learn how to use Maple through dedicated lab sessions and then are invited to use it alone and in groups during in-person lessons and lab sessions, inside a Digital Learning Environment integrated with the ACE and to solve contextualized problems connected with their future job. Moreover, as a part of the final examination, students have to study, present and discuss a science-based problem solved with Maple. They must show mastery in Mathematics underlying the problem, critical and computational thinking, proper use of Maple, ability to generalize, to justify and to make arguments for the provided solutions.

The research question of the paper is the following: did the use of Maple facilitate students to develop problem solving skills, to learn Mathematics and to understand its applications? To answer, in this research work, we examined the effectiveness of the approach adopted through the analysis of 300 students' submissions through a rubric evaluation, in relation to their final exam mark, their class, lab and online attendance. The rubric evaluates different dimensions: comprehension, resolution strategy identified, solution process, representation, argument, use of Maple. A qualitative analysis is also provided by the answers to questionnaires: students found the adopted methodology

useful in order to understand theory and to face their future study or job (median 4 over a 5-point Likert scale), however they declared that they would like to have more time to deeply understand all potentialities of Maple.

The research shows that, since problem solving skills are not officially part of the curricula of the degree programs, their development is left to Mathematics modules and to the good will of teachers who adopt new teaching approaches and methodologies. Problem solving dedicated labs with the use of Maple should be introduced, in connection also with other disciplines, not only scientific, to facilitate analysis of data, visualization, communication, interdisciplinarity and deep understanding of concepts.

Student satisfaction determinants in hybrid learning environments based on MAPLE.

Tilo Wendler (HTW Berlin - University of Applied Sciences), **Lisa Fischer** (HTW Berlin - University of Applied Sciences) and **Marcel Dux** (HTW Berlin - University of Applied Sciences).

Abstract. Mathematics is required in almost all faculties of the Berlin University of Applied Sciences (HTW Berlin). Be it the economists who learn to understand cost, profit and revenue functions, or the engineers who are starting to develop models based on sine and cosine functions, for example. In all cases, mathematics represents the underlying language. The correct use of this language enables a better understanding of complex issues in various fields.

In Germany mathematics has unfortunately become a subject of fear and remains one of the main reasons for dropping out of university. Therefore, universities need to close the knowledge gap between the mathematical knowledge learned at school and the mathematical knowledge needed in university studies [1].

By offering a specialized course for approximately 1,000 first-year students, the university wants to help to be better prepared to tackle the upcoming challenges. To meet the requirements of students at the beginning of their studies, a hybrid course had been implemented. The combination of on-campus lectures to motivate and address more difficult topics, online MAPLE exercises and tests to allow flexible learning attracted many of them. This hybrid type of implementation also meets the current expectations of students by avoiding old-fashioned teaching methods in a highly flexible world, especially from the pandemic perspective.

MAPLE incorporated in the Moebius courseware [2] was used as part of courses to educate new students in mathematics. Lectures, online tutorials and MAPLE-based online exercises are preceded by a digital placement test, so students are enabled to reflect on their level of knowledge beforehand.

Since the students of the HTW Berlin have different backgrounds regarding their pre-university education and technical affinity, it is of special interest to look at the user-friendliness from their perspective. Therefore, in this study especially the results of the student evaluation will be discussed. This is to investigate how intuitive the MAPLE-based learning environment is and to determine the success factors and best practices.

Starting with an introduction to the aim and structure of the semi-automized student assessment, the study shows how MAPLE-graded questions are incorporated in an online-based learning environment. Looking at examples, allows the authors to present benefits and challenges using MAPLE in a mathematics course from teacher's perspective. Due to the student's evaluation their experiences will be discussed in detail as well.

The authors greatly appreciate the opportunity to share experiences with the use of MAPLE in education within the community at this conference.

References

[1] R. Wurth and K. Dürrschnabel, "cosh Mathe - cooperation schule:hochschule," 10 2014. [Online]. Available: https://cosh-mathe.de/download/makV2.0neu.pdf.

[2] Digital Ed, [Online]. Available: https://www.digitaled.com/products/courseware#.

Research and Practice of college Mathematics Curriculum Innovation Project based on Maple & CT (Computational Thinking).

Guoxing Dai (Academy of Mathematical Sciences,Jiangsu University), **Rongmei Dai** (Academy of Mathematical Sciences,Jiangsu University) and **Dianc Lu** (Academy of Mathematical Sciences,Jiangsu University).

Abstract. This project is the 2019 jiangsu Higher Education teaching reform research project, aiming at carrying out university mathematics teaching reform practice under the guidance of STEM education philosophy, and exploring effective ways for STEM teaching model to cultivate international talents of applied mathematics major in Jiangsu University.

Resource construction is the key to teaching reform and in-depth development. Our project team and teachers of Acadia University jointly compiled mathematics textbooks with "Computational Thinking (CT)" (based on Maple Applet case application) to highlight the application of mathematical innovation resources.The resource content of virtual simulation is presented online in the form of 4E(Exposition, Exploration,Exercises,Evaluate).

With the aid of information network technology, teaching platform of cloud (Mōbiūs, Moodle) advantage, explore its effect in the teaching content reform, the course will focus on the external evaluation system construction of student evaluation, will be in more than one score report details, test results, the LMS response, the web browser response, not cross limits, mobile learn APP use, cross-platform, track simulation, informal learning, tracking performance and tracking real world offline learning, interactive learning, tracking adaptive learning, blended learning, tracking long-term study, tracking group learning, and other aspects of practical application.Based on the strategies and methods promoted by knowledge management, this paper researches and develops a diagnostic evaluation system for classroom learning effect of higher mathematics teaching, and explores a new mode of higher mathematics teaching combining integrated and innovative curriculum resources with the establishment of external evaluation system.

Project will highlight the mathematical characteristics of virtual experiment and application of resources is "learning in practice", the virtual model based on the mathematical principle of the teaching material that Maple, GGB supplemented by provide function of 2D&3D graphics and animation, students can interact to change the parameters of the simulation model, operation model, operation simulation, to check the update results, results in attractive and intuitive visual way to show concept, problems, and solutions.

The interactivity and visualization of resources can increase students' interest in learning. The project focuses on the innovation instant feedback mechanism, which makes up for the problem of the increasing number of students' one-on-one communication opportunities and time in the existing curriculum, which is replaced by the calculation of the thought technology and the teaching and answer of some of the field, which can improve the learning effect.

Using Maple in Humanities-Related Assessment: Exploring Beyond Mathematical Boundaries in Education.

Ananthan Ambikairajah (Australian National University), **Christopher Tisdell** (The University of New South Wales).

Abstract. Maple has been successfully implemented as an assessment tool in the mathematical sciences for years, however its educational potential in non-STEM environments remains unclear. In particular, there is a need for research to be undertaken and evidence produced to indicate if and how Maple can form a workable assessment solution for students, teachers, and administrators in non-STEM environments.

The purpose of this presentation is to respond to the above needs by exploring what roles Maple can play in the humanities via case study research. We deepen our understanding of the design, reception and effectiveness of Maple platforms employed for assessment in history and philosophy of science modules, undertaken by first-year advanced science and medical science students at a large university.

We employ a quasi-experimental research design approach to examine our implementation on reported student satisfaction regarding the suitability of the information provided about the assessment requirements, appropriateness of the assessment methods, and overall quality of the associated courses. We report statistically significant increases in student satisfaction regarding the suitability and appropriateness of the assessment methods or requirements.

The outcomes of this research highlight new avenues for educators to explore including (a) the innovative use of associated software (Maple TA™) for the humanities in non-traditional contexts and (b) the implications that electronic examinations can have on the student experience in the context of medium-stakes testing.

Maple for Distance Education in Secondary Schools during the Emergency from Covid-19.

Cecilia Fissore (Università degli Studi di Torino), **Francesco Floris** (Università degli Studi di Torino), **Marina Marchisio** (Università degli Studi di Torino) and **Matteo Sacchet** (Università degli Studi di Torino).

Abstract. As UNESCO reports, most governments around the world have temporarily closed educational institutions to contain the spread of the COVID-19 pandemic. In Italy, schools were closed in early March until the end of the school year in June. All teachers took action to facilitate the continuity of education through remote learning with the use of technological methods and tools. The beginning of the next school year is fixed for 14th September but due to the uncertainty, teachers are invited to contemplate at least blended scenarios with their students.

The context of this research is a community of teachers in disciplines like Science, Engineering, Technology and Mathematics (STEM) from different Italian secondary schools, within the Ministerial Project PP&S-Problem Posing and Solving. In this community, teachers exchange materials, ideas, and useful advices, they participate in training activities and have the constant support of expert tutors. The project involves the use of a Digital Learning Environment (DLE) for STEM, a Moodle platform integrated with Maple for the creation of interactive materials, which help the exploration of mathematical concepts and the developing of problem solving skills. Teachers used the DLE in normal teaching before the emergency and continued to use it during the emergency. Before the emergency, teachers used Maple in different ways: carrying out problem solving activities in a computer lab also in groups; making students submit their works created with Maple; projecting the interactive files in class through IBW for a theoretical explanation or for an exploratory activity;

uploading interactive materials on the DLE for asynchronous activities. During the emergency, Maple not only proved to be useful for dealing with forced distance teaching more easily but it showed to be a very convenient tool, flexible for new modalities and new purposes like synchronous online lessons, formative online assessment, adaptive activities for students with difficulties.

The research question is the following: what kind of support can Maple integrated in a DLE give for Distance Education? To answer this question, we considered teachers who used Maple through the DLE during this school year, uploading a Maple worksheet resource within one of their courses with students. We carried out a quantitative analysis on 74 courses held by 40 teachers to understand how much they had used this type of resource. On average, 27% of course materials are worksheets, ranging from 5% to 84%. There are therefore teachers who have conducted the entire course with this teaching material and teachers who have rarely used it. A qualitative analysis was then conducted to study for what purpose this educational resource was used, to understand the potential and effectiveness that Maple resources can give to STEM Education. Analysis shows that teachers have used Maple in a meaningful way to propose problem-solving activities, in-depth theoretical explanations and interactive resources for mathematical exploration. Some significant examples will be shown to explain how this type of resource can enrich STEM teaching and learning. Finally, a written interview of a group of teachers who used Maple the most was proposed. The interview aimed to understand how their way of using integrated DLE for remote teaching has changed, and the impact this has had on students. The analysis shows how teachers have studied how to redesign teaching materials to better guide students and to increase their effectiveness.

Students working SMARTER.

Alexander Rusnak (Evergreen Valley College).

Abstract. Most advanced math classes are a time tradeoff between detailed problem solutions vs. the theory. The time spent on the detailed solution can be overwhelming, leaving less time for focusing on the techniques and theory. Maple provides an excellent solution to bridge this issue, but the learning curve for can be a deterrent. However, Maple is programmable and allows for detailed programmable solutions for general types of problems. Generation of specific solutions allows students to verify solutions with minimal knowledge of Maple. This technique has been used multiple times successfully for both a Linear Algebra and a Differential Equations class. Examples used in these classes will be presented.

This solution is to provide minimal and simple formula that use Maple commands at the beginning of a problem that the student can easily modify. With the use of symbolic notation, the learning curve is minimal. Maple can compute the intermediate steps and final solution. The details of the Maple file are provided by an instructor, but usable by the students. The types of problem can range from simple tables with example formula, i.e. derivatives, anti-derivatives, or Laplace Transforms; to much more complex problem, i.e. the complete solution to a differential equation.

This technique has been used successfully in multiple classes including Linear Algebra and Differential Equations. For Linear Algebra, a file was generated and updated with the techniques as discussed in class. Students and instructor saw a significant improvement in time in the manipulation of the equations and less round-off error with detailed and readable results. For the Differential Equation class, multiple files were generated that included an editable table of Laplace Transforms, solutions to a general differential equation, and very flexible example for the Forward and Inverse FFT transforms. Using this techniques, students and instructor could try different techniques in a very minimal amount of time.

Biography: Al Rusnak received is MSEE from Santa Clara University. He has worked in the high tech industry for 35 years, with the last 20 years in semiconductor program management. He has developed a Monte Carlo simulation technique for scheduling at the product level that has been presented at PMI's Global Conference. The last few years, he has been focusing on working with instructors to help students focus on math techniques using Maple.

Maplesoft Presentation: Introducing Maple Calculator and Maple Learn

Karishma Punwani (Maplesoft)

Abstract. In recent months, Maplesoft has introduced new ways for students and teachers to leverage the power of Maple to support math education. In this talk, you'll learn about the free Maple Calculator app, which allows students to solve math problems, check homework, and explore graphs on their phones. You'll also get a chance to see a demonstration of Maple Learn, an online version of Maple designed specifically for teaching and learning math and solving math problems, from high-school to second year university.

Wednesday - Algorithms and Software

Rule-based Integration Using {Maple}.

David J. Jeffrey (University of Western Ontario), **Mahsa Kazeminooreddinvand** (University of Western Ontario) and **John May** (Maplesoft).

Abstract. Finding antiderivatives of expressions (indenite integration) is a fundamental operation in symbolic computation. Rubi [1] is a collection of roughly 7600 symbolic integration rules which can be used to calculate the antiderivatives of a wide range of mathematical expressions. The rules are published as PDF files (for humans to read) and in les written in Mathematica syntax (for computer systems to read). They are organized in a series of sub-collections, depending on the structure of the integrand. In addition, there is an extensive test-suite of over 72,000 integration problems which is utilized to evaluate each release of Rubi. Not surprisingly, when Rubi is applied to its own test suite, it performs very well. Several open-source packages ([2{4]) have incorporated pasts of the Rubi collection in their own integration routines.

The work presented here aims at translating Rubi rules from their published forms (in Mathematica syntax) into rules readable by Maple. To this end, we use string processing along with Maple's MmaTranslator package for the translation. The first step is to put the commands in a format which allows us to use Maple's patmatch command on them with additional condition checking. This leads to implementing a matcher function which takes an integrand as input and applies Rubi rules recursively to nd the antiderivative. We plan eventually to experiment using Maple's compiletable command to compile the patterns into a decision tree.

We note that each Rubi rule consists of three components: a pattern (for an input integrand), a transformation, and a set of conditions under which the first two components can be applied.

To handle the creation of patterns for Maple, we split each condition into two parts. The first part, conditions on paramters corresponding to simple Maple types, is integrated into pattern. The second part is left as an expression which should evaluate to a boolean when the matcher function gets called.

One of our eventual goals is to make a platform that can automatically compile each new Rubi release into a callable Maple command (including showing steps). We will present work that partially

achieves that goal. Our second goal is to identify places where the current Rubi depends on specic features of the Mathematica platform and to contribute back enhancements to Rubi to help make it more platform neutral.

2 Jerey, Kazemi, and May

References

1. Rich, A., Scheibe, P., Abbasi, N.,: Rule-based integration: An extensive system of symbolic integration rules. Journal of Open Source Software 3(32), 1073 (2018)

2. Expreduce, https://github.com/corywalker/expreduce

3. SymJa, https://github.com/axkr/symja_android_library/wiki/Porting-Rubi-Integration-rules-to-Symja

4. SymPy, https://github.com/sympy/sympy/tree/master/sympy/integrals/rubi

AGADE---a Maple package for computing rational general solutions of algebraic ODEs.

Johann Josef Mitteramskogler (Research Institute for Symbolic Computation (RISC), **Johannes Kepler** University Linz).

Abstract. The problem of computing exact solutions of ordinary differential equations (ODEs) is notoriously difficult in the general (non-linear) case. For algebraic ODEs, i.e. ODEs where the independent variable and the dependent function including its derivatives satisfy a polynomial relation, a new algebro-geometric approach has emerged in recent years [1]. This method operates in two stages. First, the differential equation is reduced to an algebraic problem by considering the dependent function and derivatives thereof as additional independent variables. The result defines a hypersurface which one attempts to parametrize with functions from a sought-after class of solutions. Provided such functions are found, the challenge of the second stage is to transform the parametrization in such a way that its members satisfy the differential aspect as well. This process is reasonably well understood for the class of rational general solutions, where it has led to several algorithms which may be implemented in a computer algebra system.

In this talk we will introduce the Maple package AGADE (\underline{A}lgebro-\underline{G}eometric methods for solving \underline{A}lgebraic \underline{D}ifferential \underline{E}quations) which implements several algorithms following the aforementioned approach. At present, the package offers the two methods for computing rational general solutions of first-order algebraic ODEs described in Ngô and Winkler [2] and Vo, Grasegger and Winkler [3]. Furthermore, a specialized algorithm for autonomous first-order algebraic ODEs proposed by Feng and Gao [4] is provided. In addition, an implementation for computing rational general solutions of systems of autonomous algebraic ODEs is currently under development. During the talk, we will outline the algebro-geometric method via examples and demonstrate how the package AGADE can be used to compute solutions of several algebraic ODEs.

References:

[1] F. Winkler. ``The algebro-geometric method for solving algebraic differential equations---a survey''. Journal of System Science and Complexity 32 (2019), pp. 256--270.

[2] L.X. C. Ngô and F. Winkler. ``Rational general solutions of first order non-autonomous parametrizable ODEs''. Journal of Symbolic Computation 45.12 (2010), pp. 1426--1441.

[3] N. T. Vo, G. Grasegger and F. Winkler. ``Deciding the existence of rational general solutions for first-order algebraic ODEs''. Journal of Symbolic Computation 87 (2018), pp. 127--139.

[4] R. Feng and X. Gao. ``Rational general solutions of algebraic ordinary differential equations''. Proceedings of the 2004 International Symposium on Symbolic and Algebraic Computation (2004), pp. 155--162.

The TruncatedSeries Package for Solving Linear Ordinary Differential Equations Having Truncated Series Coefficients.

Sergei Abramov (Dorodnicyn Computing Centre, FRC Computer Science and Control of RAS), **Denis Khmelnov** (Dorodnicyn Computing Centre, FRC Computer Science and Control of RAS) and **Anna Ryabenko** (Dorodnicyn Computing Centre, FRC Computer Science and Control of RAS).

Abstract. Linear ordinary differential equations with coefficients in the form of truncated power series are considered. (Thus, concerning the original equation we have incomplete information: for a power series, only a finite number of initial terms are known). First, we investigate what can be learned from the equation given in this form about its solutions in the field of Laurent formal series (we call them Laurent solutions). We are interested in the information on these solutions that is invariant under all possible prolongations of all the truncated series that represent the coefficients of the equation (the prolongation is a series whose initial terms coincide with the ones of the original truncated series). A similar question is discussed for regular solutions. In both cases, the proposed algorithms construct the maximal possible number of invariant initial terms of the series involved in the solutions.

The approach that we use in the algorithms for computing Laurent and regular solutions, allows, in combination with the well-known algorithm of Newton polygons, to construct formal exponential-logarithmic solutions of linear ordinary differential equations having coefficients in the form of truncated power series. The series which appear in the solutions have also only a finite number of known initial terms.

Linear ordinary differential equations with the coefficients which are either algorithmically represented power series, or truncated power series are considered as well. For such a mixed case, the problem of the construction of the maximal possible number of terms of the involved in the solutions series is algorithmically undecidable (for some such equations, the information is sufficient for computing any number of terms of the series). This undecidability is, so to speak, not too burdensome. If we are interested in all solutions with a truncation degree not exceeding a given integer k then the proposed algorithm allows to construct all of them.

All the proposed algorithms are implemented as the TruncatedSeries Maple package.

Puiseux Series Solutions of Autonomous AODEs.

Francois Boulier (Université de Lille - CRIStAL UMR 9189 - Inria), **José Cano** (Universidad de Valladolid), **Sebastian Falkensteiner** (Johannes Kepler University Linz, RISC Hagenberg) and **Rafael Sendra** (Universidad de Alcala, Madrid).

Abstract. Given a first order autonomous algebraic ordinary differential equation with coefficients in a computable field such as the rational numbers, we have shown a method to compute all formal Puiseux series solutions.

In fact, all of the solutions are convergent Puiseux series.

By considering the differential unknown and its derivative as independent variables, the differential equation implicitly defines an affine plane curve where (rational) local parametrizations can be computed via Puiseux expansions.

We show a sufficient and necessary condition on such a local parametrization to obtain a formal Puiseux series solution of the original differential equation by substitution.

This leads to a complete characterization of initial values with respect to the number of distinct solutions extending them.

Moreover, by choosing a particular initial value, we give an algorithm computing the coefficients of all solutions starting with this initial value up to an arbitrary order.

These results have been extended to systems of autonomous ordinary differential equations in one differential indeterminate in by using regular chains.

For several differential indeterminates some algorithmic problems occur, which can be avoided in the case of algebraic Puiseux series solutions.

In the underlying publications are algorithms of most of the theoretic results mentioned above.

We have implemented them in Maple and use already existing standard packages such as algcurves and RegularChains.

In the talk we would illustrate the performance of our implementations by means of several examples.

Blends in Maple.

Robert Corless (University of Western Ontario) and **Erik Postma** (Maplesoft).

Abstract. A *blend* of two Taylor series for the same smooth real- or complex-valued function of a single variable can be useful for approximation. We use an explicit formula for a two-point Hermite interpolational polynomial to construct such blends. We show a robust *Maple* implementation that can stably and efficiently evaluate blends using linear-cost Horner form, evaluate their derivatives to arbitrary order at the same time, or integrate a blend exactly. The implementation is suited for use with *evalhf*. We provide a top-level user interface and efficient module exports for programmatic use.

Maplesoft Presentation: Our Favorite Things: Maple 2020 Gems You May Have Missed

Samir Khan and Karishma Punwani (Maplesoft)

Abstract. Every new Maple release contains many improvements, large and small. While the large changes get the most attention, there are always many small improvements that turn out to be really useful. In this talk, you'll learn about some of our Product Managers' favorite unglamorous things in Maple 2020. You may even get a sneak peek at some of our new favorites for 2021!

Multivariate Power Series in Maple.

Mohammadali Asadi (The University of Western Ontario), **Alexander Brandt** (The University of Western Ontario), **Mahsa Kazemi** (The University of Western Ontario), **Marc Moreno Maza** (The University of Western Ontario) and **Erik Postma** (Maplesoft).

Abstract. In this work, we implement multivariate power series in Maple as the MultivariatePowerSeries package, providing two Objects: PowerSeriesObject, and UnivariatePolynomialOverPowerSeries (UPoPS). This package provides lazy arithmetic for power series and UPoPS, most notably, with an optimized implementation of Weierstrass preparation theorem for UPoPS, and factorization of UPoPS based on Hensel's lemma.

Taylor Series Solutions of Delay Differential Equations Using dsolve solver in Maple.

Samir Hamdi (Department of Computer Science, University of Toronto).

Abstract. The Maple solver dsolve can handle different types of Ordinary Differential Equations (ODES). It can be used for symbolic computation of ODE solutions but also for computing numerical solutions for given initial conditions (boundary value problems) using dsolve/numeric. Taylor series solution of ODEs can also be obtained using the option dsolve/series. The series solution in dsolve is based on Geddes's method. The basic algorithm is a Newton iteration, which consists of considering a bivariate Taylor series expansion of the ODE and solving the resulting linear first order ODE at each iteration. The application of the Newton iteration almost doubles the number of correct coefficients of the Taylor series solution. In this paper we first implement Geddes's algorithm in Maple for the solution of ODEs and we provide several examples for illustrations.

In the second part we apply Geddes's algorithm for Taylor series solutions of Delay Differential Equations (DDEs) using Newton iteration. This extends the capabilities of dsolve/series to handle delay equations and allows the user to specify the history function for delay equations instead of initial conditions. Several worked examples with documented Maple codes are provided to demonstrate how to solve delay differential equations with Taylor series using dsolve/series. Illustrated examples and test sets are considered from published articles and reports and include delay differential equations with constant lags, time dependent delays, state delays and small/vanishing delays, and also delay differential equations of neutral type.

Thursday - Applications of Maple

Modelling and sensitivity analysis of nonlinear firefighting systems using Maple.

Flóra Hajdu (Széchenyi István University, Department of Mechatronics and Machine Design), **Győző Molnárka** (Széchenyi István University, Department of Mechatronics and Machine Design) and **Rajmund Kuti** (Széchenyi István University, Department of Mechatronics and Machine Design).

Abstract. This presentation gives an overview of the modelling of nonlinear firefighting systems using Maple. In the presentation the construction of mathematical models, their implementation to Maple and numerical simulation results are explained in detail. The examined models include different degrees of freedom heavy-duty fire truck suspension models, a double-cabin fire truck suspension model and elements of a firefighting system. Furthermore detailed One-at-a-Time sensitivity analysis results are also presented using a novel fuzzy-logic based evaluation method developed in Maple. Different possibilities of parameter estimation using the method are also discussed. The presentation concludes with further research tasks.

Using Maple to solve real-time scheduling problems.

Jean-François Hermant (ESIEE Paris).

Abstract. This paper is devoted to the study of real-time scheduling problems that arise when considering a processor.

Tasks are characterized by their worst-case execution times (C), periods and/or sporadicity intervals (T), and relative deadlines (D) and are scheduled on the processor according to a real-time scheduling algorithm.

We consider Fixed Priority-driven scheduling algorithms (Highest Priority First with priorities computed à la Rate- or Deadline-Monotonic) and Deadline-driven scheduling algorithms (Earliest Deadline First) in the preemptive case.

To solve a real-time scheduling problem, for a given task set and for a given scheduling algorithm, we have to establish feasibility conditions, i.e. to provide a set of constraints that captures the dynamic behavior of the algorithm and that allows us to know whether the task set is feasible or not, i.e. if all the task deadlines are met or not. These feasibility conditions can be necessary, necessary and sufficient, or sufficient.

Among the infinite set of all the possible activation scenarios, we have to identify a finite set of activation scenarios, called the worst-case activation scenarios, such that, if the task set is feasible on this finite set of worst-case activation scenarios, i.e. if no task deadlines are missed, then the task set is also feasible on the infinite set of all the possible activation scenarios.

The first part of this paper focuses on the Maple implementation of the feasibility conditions for Fixed Priority-driven scheduling algorithms (HPF/RM or HPF/DM) and Deadline-driven scheduling algorithms (EDF) when all the task parameters (C, T, D) are known and fixed. The necessary, necessary and sufficient, and sufficient feasibility conditions for these algorithms are implemented on Maple. The implementation is straightforward and provides an efficient dimensioning tool.

The second part of this paper focuses on the Maple implementation of the necessary and sufficient feasibility condition for EDF when the task worst-case execution times (the C's) are unknown and when the other task parameters (the T's and the D's) are known and fixed. We show that the feasibility domain for EDF is convex, which is directly related to the optimality of EDF. The simplex algorithm is used to eliminate all the redundant constraints and to provide an efficient and elegant computation of the feasibility domain for EDF.

These results can be used to compare Fixed Priority-driven scheduling algorithms (HPF/RM or HPF/DM) and Deadline-driven scheduling algorithms (EDF) in terms of efficiency.

Numerical solution for radial distortion rectification in optical systems.

Obed Isai Rios-Orellana (Instituto Politecnico Nacional), **Rigoberto Juarez-Salazar** (Instituto Politecnico Nacional) and **Victor Hugo Diaz-Ramirez** (Instituto Politecnico Nacional).

Abstract. Proper homography estimation is a crucial step in many applications in computer vision. Nevertheless, nonlinear optical properties in cameras such as the wide-angle lens may introduce radial distortion, making unsuitable apply the pinhole model for homography estimation. In this paper, we propose an algorithm to rectify radially distorted images using Maple. We initially analyze the model that produces the radial distortion effect and then an inverse distortion model is proposed. This proposal allows us to estimate the homography matrix and distortion parameters by processing images of a calibration target using the Gauss-Newton approach. We illustrate with an example the usefulness of the proposed method. The proposed method is feasible for successful estimation of homography matrices and distortion parameters to correct real world images

Maplesoft Presentation: Generate Captivating Visualizations with Maple

Valerie McKay-Crites (Maplesoft)

Abstract. Regardless of the domain of your calculations and data; visualizing your results is what brings them to life. Not only does Maple allow you to calculate and visualize your results all in one place, it allows you to do it easily, elegantly, and professionally.

This talk is for you if you want to create illuminating visualizations that help students grasp concepts and make your papers pop out and easy to follow for readers. Using Maple, you can create visuals that will help the viewer to really understand the meanings your data shows.

While you're probably familiar with the basics of plotting in Maple, there is so much more you can do to elevate your results and really captivate your audience.

Maple as a Tool in HTCondor-managed High-Throughput Computing Environments.

Rémi Stellian (Pontificia Universidad Javeriana).

Abstract. We present a method for preparing the execution of Maple worksheets in a High-throughput Computing (HTC) environment managed by HTCondor. The software HTCondor creates an HTC environment that exploits the computing power of a cluster of machines. This kind of environment is particularly relevant when the same Maple procedure is run many times, perhaps with differences in parameters or in the random objects generated by the procedure. Such an HTC environment also provides a solution for running a collection of procedures sharing common statements. In both cases, the HTC environment enables the execution of a large amount of Maple worksheets on machines connected over a network. HTCondor provides a framework to assign multiple tasks –or ``jobs''-- to different machines taking into account available RAM memory, CPU type, CPU speed, and current load average, among other properties.

Submitting jobs to an HTCondor-managed cluster of machines requires the creation of text files that control the details of submission. Text-based MPL files that contain the Maple procedures to be run must also be created. We show how the subpackage Text of the package FileTools is helpful for creating these two sets of files. We provide several illustrations of how a collection of procedures can be incorporated into a set of HTCondor jobs whose runtime is neither too short nor too long.

The presentation is structured as follows. First, we introduce the basics of running a job in HTCondor. Second, we give examples of Maple procedures that could be run on an HTCondor-managed cluster of machines. Third, we explain and illustrate the use of FileTools[Text] to create the files necessary for running jobs in HTCondor. Lastly, additional considerations on how other Maple commands can enhance the efficiency of computing power use under the HTCondor environment are given.

Estimation of Travel Times for Additional Metrobus Route.

Verónica Nieves (UNAM) and **Patricia E. Balderas** (UNAM).

Abstract. In this work we present some results of a research with application case study format of a simulation model, as a tool that supports decision making, developed in Coyocan, Mexico City, where an extension is proposed for an existing line of the Metrobus System. The simulation model is developed with Maple tools. The main objective was to estimate the travel time for the extension of the Metrobus line proposed, considering the time traveled between two adjacent stations, the waiting time at each station (intended for passengers to board and leave the units), and the waiting time at intersections with traffic lights. The simulation model takes as input parameters the probability functions that characterize the operating speed of the units on a current Metrobus line, the duration of each phase of the traffic light cycles and the distance between stations and traffic lights, within the proposed route. With these parameters, random variables are generated that represent those durations and are subsequently added to estimate the total duration of the trip. The model allows the user to decide the number of replicas (r) and the amount of data in each one (d), in such a way that the results show the graphs corresponding to the average value at each replica and the confidence interval of this parameter. Finally, the results are compared with the duration of trips that are currently made in trucks and minibuses as rationale of the proposal.

Thursday - Maple in Mathematics

Conway Type Generalizations of The 3x+1 Problem and p-adic Representation of Rational Numbers.

Yagub Aliyev (ADA University).

Abstract. The 3x+1 Problem is one of the longstanding problems in mathematics [2]. It is about the following simple algorithm. Take a positive integer. If it is even then divide it by 2, otherwise i.e. if it is odd then multiply the number by 3 and add 1. Continue to do the same with the resulting number. For example if you start with 7, then the next numbers are 22, 11, 34, 17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1, 4, 2, 1, ... As one can observe from this example the algorithm reaches 1. It is still unproven that for any starting number, eventually, there is a step where the algorithm reaches 1. The other observation is that after 1 we obtain a periodic se-quence 4, 2, 1, 4, 2, 1,... Again, it is still unproven that it is the only such possi-ble period for this algorithm. In the current paper, a new approach for the second problem will be discussed. Instead of choosing a starting number and letting it dictate what operations are applied to the following numbers, a fixed sequence of Conway [1] type operations are chosen and the starting number, which returns to itself when the given operations are applied to it in the given order, is determined using these operations. Because of this change in the perspective, the starting number and the numbers which follow it are, in general, not positive and not in-tegers anymore. The obtained rational numbers enjoy some interesting properties, discussion and proof of which is the main focus of the paper. These properties can also be expressed in terms of p-adic numbers.

An Isometric Embedding of the Impossible Triangle into the Euclidean Space of Lowest Dimension.

Zhenbing Zeng (Department of Mathematics Shanghai University), **Yaochen Xu** (Department of Mathematics Shanghai University), **Zhengfeng Yang** (Shanghai Key Lab of Trustworthy Computation, East China Normal UNiversity, Shanghai 200062, China) and **Zhi-Bin Li** (School of Computer Science and Technology, East China Normal University).

Abstract. The impossible triangle, invented independently by Oscar Reutersvard and Roger Penrose in 1934 and 1957, is a famous geometry configuration that can not be realized in our living space. Many people admitted that this object could be constructed in the four dimensional Euclidean space without rigorous proof. In this paper, we proved that the isometric embedding problem can be decided by finite points on the configuration, then applying Menger and Blumenthal's classi classical method of Euclidean embedding of finite metric space we determined the lowest Euclidean dimension, and finally using the symbolic algebraic computation we obtained the coordinates of the isometric embedding.

Our investigation shows that the impossible triangle is impossible to be isometrically embedded in the four dimension Euclidean space, but there is an isometric embedding of the impossible triangle to the five dimension space.

Computation of the L_∞-norm of finite-dimensional linear systems.

Yacine Bouzidi (Inria Lille), **Alban Quadrat** (Inria Paris, Ouragan project, IMJ -- PRG, Sorbonne University), **Fabrice Rouillier** (Inria Paris, Ouragan project, IMJ -- PRG, Sorbonne University) and **Grace Younes** (Inria Paris, Ouragan project, IMJ -- PRG, Sorbonne University).

Abstract. In this paper, we study the computation of the L_∞-norm for finite-dimensional linear systems. This problem is first reduced to the computation of the maximal *x*- projection of the real solutions (*x*, *y*) of a bivariate polynomial system $\{\mathcal{P}, \frac{\partial \mathcal{P}}{\partial y}\} \subset \mathbb{Z}[x, y]$. We then apply computer algebra methods to solve the problem. We alternatively study a method based on rational univariate representations, a method based on root separation, and finally a method based on the sign variation of the leading coefficients of the signed subresultant sequence and on the identification of an isolating interval for the maximal *x*- projection of the real solutions of the system.

Maplesoft Presentation: Analytic Approximation for the Dirichlet Problem

Dr. Robert Lopez (Maplesoft)

Abstract. The 1964 Benster translation of the Kantorovich and Krylov text "Approximate Methods of Higher Analysis" details a scheme for approximating the solution of boundary value problems for homogeneous elliptic equations on plane regions. One example (Laplace's equation on a square) is laboriously provided, the calculations having been done "by hand."

In this webinar, the recipe given by Kantorovich and Krylov is implemented in Maple, and applied to the text's example and to several others. The ease with which the requisite calculations can be executed is the message "between the lines." The overt message is that the method works, and gives reasonable accuracy for solutions to problems that would otherwise have to be obtained numerically.

Local analysis of separatrices around saddle points in autonomous two-dimensional dynamic systems.

Luis Sainz de Los Terreros (Universidad Politécnica de Madrid (UPM)) and **Antonio Rodriguez Mesas** (Universidad Politécnica de Madrid (UPM)).

Abstract. Separatrices of saddle points are among the more important paths in the qualitative study of dynamic systems in the plane. These singular orbits divide the phase plane into regions which show similar orbital behaviour. Maple already offers several recourses for the analysis of the phase portrait of a plane autonomous system. In this document a method is presented to compute the local Taylor series expansions of separatrix curves that arises from or ends on a saddle point. A sequence of operators are defined recursively and used to compute the Taylor series coefficients of the separatrices around each saddle point. L'Hopital's is the key rule for the construction of the operators that generates the series coefficients.

A number of cases are presented, from the classical one of the pendulum to other conservative and Hamiltonian dynamic systems of physical relevance. Systems with heteroclinic orbits are also considered and even dissipative dynamical systems. A last example of a system with a global bifurcation which depends upon the value of a parameter is presented and discussed. All the examples are accompanied with plots that shows the usefulness of the method at the graphical level.

Using Maple to deal with offsets to quadrics.

Laureano Gonzalez-Vega (Universidad de Cantabria), **Jorge Caravantes** (University of Alcalá), **Mario Fioravanti** (Universidad de Cantabria) and **Gema M. Diaz-Toca** (University of Murcia (Spain)).

Abstract. Offsets to surfaces have many important applications such as tool path generation, NC milling, design of thick curved surfaces and tolerance analysis. Given a rational surface, its offset is an algebraic surface, generally not rational and bringing high degrees, many terms and huge coefficients. We will show how Maple was used to derive a new presentation for the implicit equation of offsets to quadrics and how it can be used to deal with intersection and visualisation problems.

A Maple Solution to the Problem 6 of the IMO 1988.

Zhenbing Zeng (Shanghai University Department of Mathematics), **Yong Huang** (Guangzhou University South China Institute of Software Engineering), **Yaochen Xu** (Shanghai University Department of Mathematics), **Xiaoru Chen** (Guangzhou University South China Institute of Software Engineering) and **Lu Yang** (Chengdu Institute of Computer Applications Chinese Academy of Sciences).

Abstract. We will report a Maple experiment for finding a solution to the Problem 6 of International Mathematical Olympiad 1988, which asks to prove that if integers a and b satisfy that ab+1 devides a^2+b^2, then 他 the quotient (a^2+b^2/(ab+1) must be a square of integer number. Only 11 people among 248 contestants answered the problem in that year's competition, so the problem was considered as a rather difficult one. We will start from a very short Maple program, then find some idears from analysing the data produced by Maple program, and show that it would suffice to write down a strict solution to the Problem 6. In the final section section of our paper will show how to reconstruct polynomials generalized from certain recurrence formula bu searching patterns from the Yang Hui's (i.e., Pascal's) Triangle.

Friday - Maple in Education

Using Maple to Teach the SIR Model.

Douglas B. Meade (University of South Carolina - Columbia).

Abstract. In 2001, David Smith and Lang Moore published "The SIR Model for Spread of Disease" in the MAA's Journal of Online Mathematics and Its Applications. Their online teaching materials in the MAA's MathDL can be found at https://www.maa.org/press/periodicals/loci/joma/the-sirmodel-for-spread-of-disease-introduction. Their materials include the development of the classical Kermack-McKendrick SIR model, understanding the parameters in the model, and emphasizing the significance of the contact number and herd immunity. All of this is done at a level understandable by a student in first-semester calculus.

A lot has happened since 2000. In this talk I will present ways in which I have included this application in my calculus and differential equations classes. I will show how I have updated the content for COVID-19 and other infectious diseases. I will also show how I have updated the ways in which recent additions to Maple are used to increase student involvement and understanding.

Quantum Chemistry Toolbox in Maple for Research and Education.

Jason Montgomery (Florida Southern University) and **David Mazziotti** (University of Chicago).

Abstract. The Quantum Chemistry Toolbox (QCT) in Maple provides an integrated environment for the computation and visualization of the quantum energies and properties of molecules. In this lecture we will highlight the advantages of QCT for computational chemistry research and education. While the QCT contains standard methods like density functional theory (DFT) and time-dependent DFT for ground and excited states, it also contains modern two-electron reduced density matrix (2-RDM) methods—in which the 2-RDM is computed directly without construction or storage of the many-electron wave function—for treating strongly correlated molecular systems. The computational methods in the QCT are accessible in the integrated scientific environment of the Maple worksheet which allows the user to create professional documents with a combination of molecular computations, chemical data, 2D and 3D plots, animations, and pictures. The QCT shares Maple's powerful but high-level programming language and its ability to export its data, plots, and pictures to LaTeX, Excel, Python, Matlab, PDF, PNG, and many other programs, languages, and formats. The computational methods, fully featured interface, and connectivity make the QCT a powerful and modern tool for computational chemistry research with applications from synthetic and physical chemistry to materials research. Moreover, the QCT and Maple provide an ideal pairing for the teaching of chemistry concepts through computational labs, demos, and exercises in undergraduate and graduate courses. While historically chemistry courses have required students to learn and use multiple programs such as a quantum chemistry code and a computer algebra system like Maple to teach different aspects of computation, the QCT provides built-in lessons covering the full spectra of computational lessons. A selection of lessons will be presented, highlighting the versatility of QCT as an educational resource for faculty and students.

An Early Introduction to the Frenet-Serret Frames in Calculus III: A project-based approach using Maple.

Lancelot Gooden (Johnston Community College).

Abstract. During this talk, I will demonstrate the Maple application supplied to students as a guide in Calculus III to solve various projectile motion problems in space as a semester group project. The app also required derivation of the kinematic equations, expressed as position vectors, using first-order linear differential equations completed entirely in Maple. The purpose of this project-based approach to studying projectile motion in space was to enhance students' knowledge of vectors in the plane and space and vector-valued functions while providing an early introduction to curve geometry, specifically the Frenet-Serret Frames. The related lessons taught leading up to this project allowed students to demonstrated an in-depth understanding of the applications of vector operations and Vector-Valued Functions in 3D while enhancing their mathematics and technology literacy in the Maple learning environment.

In document mode, students used function definitions, line commands, plotting commands, and animation features to produce a dynamic solution to the motion of a particle in space that has undergone forces due to acceleration. Additionally, students eventually imposed the Unit Tangent, Principal Unit Normal, and Binormal Vectors (Trihedral) bounding the Frenet-Serret Frames, to the particle in motion. By the end of this project, students would have acquired a solid foundation to study further the relationship between the trihedral vectors, torsion, curvature, and the Frenet-Serret equations.

The Maple learning environment made the execution of this project manageable as an instructor and for students while providing the appropriate level of rigor for students in Calculus III.

Understanding math concepts in music and vice-versa.

Gabriel Picioroaga (University of South Dakota).

Abstract. The aim of this presentation is pedagogical in nature and two-folded. It is primarily intended to instructors who teach undergrad math courses (such as Lineal Algebra, Calculus or Fourier Analysis), or courses in basic Music Theory.

We show how to use the AudioTools and SignalProcessing in Maple to illustrate and explain at a deeper level the mathematics needed to implement and play the following musical concepts: tones, overtones, chords, and more complex paradigms such as "counterpoint", "horizontal thirds", "vertical fifths".

Starting with a fundamental frequency f0 (the classic 440 Hz for the note A on the piano) all musical pitches are encoded in Maple as sine waves using the nice logarithmic formula between pitches and frequencies (in musical terms, we have implemented the "temperate tuning"). Hence all pitch classes are associated with the numbers 0 =C, 1=C#, 2=D,..., 11=B, 12=C an octave higher, and so on. Our procedure Note(i,t,x) encodes the pitch i, played for t seconds, at amplitude x. A melody then becomes a sequence (array) of notes that can be played either straightforward and/or saved as a .wav file.

For advanced math courses one can implement the convolution and (inverse) Fourier transforms acting on audio files represented as arrays, in order to apply special sound effects. With a little creativity, one can come up with a tremendous number of projects that may prove appealing to both math and music majors. In this fashion we can show our students that many math concepts (just

think of how hated trig functions are, undeservingly so) have easy and striking applications in world more tangible than an abstract realm.

We believe that a few lectures based on the ideas above, may bring important education benefits to a large spectra of undergraduate students taking math/music courses, regardless of their major.

The Use of Maplesoft in an Honors Calculus Course.

Matthew Westerhoff (Northern Virginia Community College).

Abstract. With the ever-growing presence of technology, it is becoming more important to incorporate technology into STEM courses as educational tools for students. In mathematics it is not only important to know how to solve calculations by hand but also, it's just as important to know how to use a computational software tool such as Maple to implement and solve problems. Not only can Maple be used to render visuals, but it can provide students the opportunity to use a commercially licensed computational tool while learning the concepts of a given mathematical topic. In this paper I will present a sample survey of assignments that I use in my honors calculus courses. Towards the end of this paper is a case study where two of my calculus students worked on a possible application of the Brachistochrone Curve as a requirement for their honors project.

Maple at TAMU SEE-Math.

Philip Yasskin (Texas A&M University).

Abstract. Since 2002, Texas A&M University has sponsored its Summer Educational Enrichment in Math (SEE-Math) program for gifted middle school students. See

http://see-math.math.tamu.edu/.

Every year, a central activity has been the creation of computer animations using Maple. You can see these animations by going to the website and clicking on each year of the archives. There are lots of programs which make animations more easily using drag and drop methods, but Maple has the advantage of actually teaching Math and programming. Students learn coordinates in the plane, circles, ellipses, translations, rotations, reflections and scaling, as well as plotting, do loops and procedures.

Over the years, Maplesoft has been an enthusiastic supporter of the program, providing copies of Maple as prizes to the animation winners. This year, the COVID-19 pandemic required us to take the program online. This allowed us to attract students on a national basis, instead of the usual local basis in driving distance from Texas A&M. About one third of the students were super-gifted, including one eight year old who already knows calculus and JavaScript programming. Maplesoft agreed to provide free copies of Maple 2020 to all students for two months surrounding the program. This allowed the students to work on their animations at home and also proved useful in teaching other activities such as cryptography and linear programming. We very much want to thank Maplesoft for their support.

Maple Interactive Texts and Simulations for Infectious Disease Modeling Projects.

John Pais (Ladue Horton Watkins High School).

Abstract. Maple interactive texts and simulations were developed, during the COVID-19 distance learning term, for a capstone mathematics project in which each student team writes an Integrative

STEM scientific paper on infectious disease modeling. Starting with these Maple interactive texts and simulations for several linear and nonlinear models, each paper must explain the mathematics for each type of model, research current infectious diseases to model using actual data, choose modeling technology, and conclude which type of model is best. In addition, each student team must create a video presentation of their work for a virtual conference.

These Maple interactive texts and simulations include an introduction to eigenvalues, eigenvectors, matrix inverses, matrix exponentials, systems of linear and nonlinear differential equations, drug absorption models, SIR and SEIR infectious disease models, the LambertW function, parameter estimation, goodness of fit, and curve fitting.

A novel aspect of this work is that Maple components with startup code are used to create fully interactive simulations for student exploration of all infectious disease model features, and also combined with separate autoexecutable code for parameter estimation in a given dataset, which can then be seamlessly used to create a new model with the dataset superimposed on the display of this model.

Friday - Algorithms and Software

Sparse polynomial interpolation and computing roots of polynomials over prime fields.

Michael Monagan (Simon Fraser University).

Abstract. The most expensive step in the Ben-Or/Tiwari method is to compute the roots of a polynomial L(z) in Fp[z] of degree t. By construction, L(z) has t distinct roots in Fp. The most well known method to do this is the Cantor-Zassenhaus algorithm from 1981. It is one of the earliest probabilistic algorithms. It is implemented in Maple. It does, on average, O(M(t) log t log p) arithmetic operations in Fp where M(t) is the cost of multiplying two polynomials of degree at most t. All other steps in the Ben-Or/Tiwari interpolation method do O(M(t) log t) or fewer arithmetic operations. In 2015 Grenet, van der Hoeven and Lecerf devised a beautiful new method which they call the ``Tangent Graeffe Algorithm''. It is also probabilistic. Note, it is not a general method for root finding in Fp[z] as it assumes p is a smooth Fourier prime. The expected running time is O(M(t) log p) arithmetic operations in Fp which improves on Cantor-Zassenhaus by a factor of O(log t). But log t is not very big. For t=1,000,000, log_2 t is only 20. Question: How much faster, if any, is the Tangent-Graeffe method than the Cantor-Zassenhaus method in practice? We have implemented the new method in C using asymptotically fast polynomial arithmetic and we have found that it is a lot faster than Cantor-Zassenhaus. So now we want to use it in Maple. We need to be able to call a C program from Maple. How can we do this? In the talk I will present the main idea behind the new Tangent-Graeffe algorithm which is the Graeffe transform. I will present some timings comparing the Tangent Graeffe algorithm with the Cantor-Zassenhaus algorithm to show how good it is in practice. I will explain how to get Maple to call a function in a C library. I will also say why sparse polynomial interpolation is a potentially important tool for Maple.

The Inverse Gamma Function and its Numerical Evaluation.

Ana Carolina Camargos Couto (Western University), **David Jeffrey** (Western University) and **Robert Corless** (Western University).

Abstract. Computing the functional inverse of the gamma function requires good starting guesses. For the principal branch, one can use an approximation generated by reversing Stirling's original

series. For other branches, it can be more difficult. We will present a thorough analysis of the complex inverse gamma function, and introduce a numerical method for evaluating the inverse gamma of a complex variable within any section of its multi-valued domain.

A Maple implementation of the finite element method for solving boundary problems of the systems of ordinary second order differential equations.

Alexander Gusev (Joint Institute for Nuclear Research), **Sergue Vinitsky** (Joint Institute for Nuclear Research, Dubna), **Vladimir Gerdt** (Joint Institute for Nuclear Research, Dubna), **Ochbadrakh Chuluunbaatar** (jinr), **Galmandakh Chuluunbaatar** (Joint Institute for Nuclear Research), **Le Hai Luong** (Ho Chi Minh city University of Education, Ho Chi Minh city) and **Eugene Zima** (Wilfrid Laurier University, Waterloo, ON).

Abstract. We present a new algorithm of the finite element method implemented as KANBP 5M code in MAPLE for solving the boundary value problems of the system of ordinary second order differential equations with continuous or piecewise continuous real or complex-valued coefficients. The desired solution in a finite interval of the real-valued in the independent variable subject to homogeneous boundary conditions: Dirichlet and/or Neumann, and/or third kind. Discretization of the boundary problems are implemented by the finite element method with the interpolation Hermite polynomials preserves the property of continuity of derivatives of the desired solutions [1]. Solutions of algebraic problems are performed using the built-in procedures of the linear algebra.

For the reduction of the boundary-value problem or the scattering problem with a different number of open channels in the two asymptotic regions to the boundary problems on a finite interval, the asymptotic boundary conditions for large absolute values of the independent variable are approximated homogeneous boundary conditions of the third kind. The program calculates the energy eigenvalues or the scattering matrix composed of square matrices amplitude reflection and rectangular matrices of transmission amplitudes, and corresponding wave functions.

For the calculation of metastable states with complex eigenvalues of energy, or to solve the problem for bound states with boundary conditions depending on the spectral parameter the Newtonian iteration scheme is implemented [2].

Benchmark examples of using of the code for solving boundary-value problems and scattering problems of quantum mechanics are given.

[1] A.A. Gusev et al, Lecture Notes in Computer Science 8660, 138-154 (2014).
[2] A.A. Gusev et al, Lecture Notes in Computer Science 9301, 182-197 (2015).

Quantifier Elimination and projection & lifting Cylindrical Algebraic Decompositions in the QuantifierElimination Package in Maple.

Zak Tonks (University of Bath).

Abstract. Quantifier Elimination over the Reals (QE) is the problem of eliminating quantified variables from a boolean formula of real polynomial constraints. QE problems arise from a range of areas including economics and biological modelling. The development of a new package for Maple QuantifierElimination is the focus of a project supported by Maplesoft researching algorithms in this area. In particular the main point of research is a poly-algorithm between two traditional algorithms for QE, Virtual Term Substitution (VTS) and Cylindrical Algebraic Decomposition (CAD). The intention is to cover the shortcomings of VTS (limitations on degree of quantified variables in

the input formula) with benefits of an incremental approach on CAD exploiting convenient nuances of usage of VTS. A further aim of QuantifierElimination is that it offers full incrementality for every methodology offered for QE, as is often desirable such that for example it could be used within a Satisfiability Modulo Theories (SMT) framework for the theory of real arithmetic. Unlike VTS, CAD is complete for QE, being able to handle a formula with polynomials of any degree. Additionally, there are a wealth of contributions from research, including bespoke optimisations for the case of QE to mitigate the doubly exponential complexity of the process. CAD is not only an algorithm for QE, but is of interest in other unquantified contexts such as motion planning. Hence it is often implemented, including in Maple. Maple already includes the RegularChains package providing an implementation of CAD. That implementation is unique in approach in the sense that it first constructs in complex space before refining to a real CAD. QuantifierElimination's CAD is a more traditional projection & lifting CAD, but gathers all contemporary research in this area, in particular very recent research with respect to the Lazard projection with optimisations via equational constraints. Hence QuantifierElimination is the first investigation as to the efficacy of such a projection & lifting CAD using these optimisations. CAD in QuantifierElimination especially uses various routines from other standard Maple packages under the hood, introducing opportunities for further development of these packages. The CAD implementation in QuantifierElimination is designed to offer output that is easily examinable for experienced geometers to investigate problems. This talk is intended to provide an overview of the QuantifierElimination package and the research behind its development, with a focus on what the package offers Maple, and opportunities for further research and development.

Factoring a Multivariate Polynomial Represented by a Black Box - A Maple to C Implementation.

Tian Chen (Simon Fraser University) and **Michael Monagan** (Simon Fraser University).

Abstract. Multivariate polynomial factorization algorithms for factoring a polynomial in Z[x1,...,xn] in sparse representation have been implemented in Maple and other computer algebra systems. However, factoring a polynomial represented by a black box can be advantageous in some cases, e.g. computing the determinant (in its factor form) of a matrix A with multivariate polynomial entries. The reason is that in general the factors have much smaller number of terms than the polynomial, we save the cost of evaluating the polynomial as well as memory space to represent it.

Previously, Diaz and Kaltofen developed FOXBOX that was written in C++. PROTOBOX developed by Wen-Shin Lee was written entirely in Maple. Our goal is to provide a user-friendly black box factorization tool that allows the users to work in Maple. However, the subroutines can be computed using C to speed up the computation. We also propose a new algorithm that outputs the factors in the sparse representation directly. Instead of first evaluating the factors and then doing a sparse interpolation (Kaltofen and Trager (1990)), we can modify and use the sparse Hensel lifting algorithm (Chen and Monagan (2020)) which is highly parallelizable. We explore both approaches and give a detailed complexity analysis and experiments implemented in Maple.

Detecting Invertible Linearizability Using the MapDETools Library.

Zahra Mohammadi (Western University) and **Gregory Reid** (Western University).

Abstract. In previous work, we developed algorithms for determining the existence of an invertible map of nonlinear differential polynomial systems and some times constructing the map if it exists [3,4]. Since many methods are known for linear systems in contrast to nonlinear systems such mappings are very useful.

Most differential equations that arise in applications are nonlinear systems and are not usually linearizable. Consequently developing efficient tests to reject the existence of mappings based on structural and dimensional information is important. An important question is how to facilitate the search for linearizable differential systems. The MapDETools package introduces a powerful algorithmic calculus for determining the linearizability based on analyzing infinitesimal symmetry features of differential equations. It determines the existence of invertible mappings of less tractable systems of differential equations (e.g., nonlinear) into more tractable systems of differential equations (e.g., linear). It also characterizes features of the map if it exists. The key idea is to apply a finite number of differentiations and eliminations to the infinitesimal symmetry systems to yield them in the involutive form, where the properties of Lie symmetry algebra can be explored readily without solving the systems. The LAVF package and the rif command improve the implementation of the MapDETools.

Simplification in Maple: Lambert W.

David J. Jeffrey (University of Western Ontario), **Toufic Ayoub** (Western University) and **Kishore Basu** (University of Western Ontario).

Abstract. A number of popular mathematical problems can be solved in terms of the Lambert W function. Often, the solution cannot be simplified exactly beyond Lambert W, and in such cases, a floating point evaluation is the only possibility. Mathematicians, however, are always interested in the special cases in which an exact simplification is possible.

We point out that chasing all such simplifications can be never ending, and one has to select which simplifications to implement.

We have looked at simplifications connected with a famous problem associated with the names Daniel Bernoulli, Goldbach and Euler. The problem is the solution of x^y=y^x.

We show that this leads to a simplification problem for Lambert W, and we describe our algorithm and its implementation.

A Maple implementation of FFT-based algorithms for polynomial multipoint evaluation, interpolation, and solving transposed Vandermonde systems.

Kimberly Connolly (Simon Fraser University).

Abstract. Suppose that we have a polynomial f(x) of degree n-1 in F[x]. There are three main problems we are interested in. The first is multipoint evaluation where, given n arbitrary points, u1,u2,...,un, we compute f(ui)=vi for i from 1 to n. Next is interpolation where, given f(ui) = vi for i from 1 to n, we reconstruct the polynomial f. Finally, we have solving a transposed Vandermonde system of equations, VTb=v, for the unknown coefficient vector b. The need to solve transposed Vandermonde systems arise in sparse interpolation algorithms, such as in the Ben-Or and Tiwari algorithm from 1988.

Classically, algorithms for polynomial multipoint evaluation, interpolation and solving transposed Vandermonde systems over a field have a quadratic running time. Horner's method may be used n times to evaluate a polynomial at n points and this costs a quadratic number of arithmetic operations in F. Interpolation has traditionally been performed using Lagrange interpolation or Newton interpolation which both need quadratic time. The system of linear equations VTb=v can be solved naively in cubic time and quadratic space using Gaussian elimination, however, Zippel showed in 1990 how to solve VTb=v in quadratic time and linear space.

More efficient algorithms for the three problems have been discovered, which are based on the Fast Fourier Transform (FFT) and require O(n log2 n) arithmetic operations in the field, but these algorithms are described separately in different literature. All three algorithms use the same background tools, mainly a subproduct tree as well as fast multiplication in F[x] with the FFT and fast division in F[x] with Newton iteration. To aid in understanding and help with clarity, we consolidated and examined these fast algorithms all in one place. We presented the algorithms, analyzed their complexity, and showed that they can be implemented in Maple efficiently.

For the talk, I will describe the subproduct tree algorithm, explain the fast multipoint evaluation algorithm and show the complexities for both. Then I will present Maple timings for the prime field Fp that compares the fast evaluation algorithm versus calling Maple's Eval(...) mod p command n times. The interesting parts are the point at which the fast code beats Maple's quadratic method and that the Maple code is quite short.

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