Chaos in Interacting Oscillators: A Classical and Bohmian Perspective

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In this talk, we begin by discussing the mechanisms through which chaos arises in classical Hamiltonian systems, as well as in their Bohmian quantum counterparts. Our aim is to understand the similarities and differences in the manifestation of chaotic behavior in these two frameworks.

To explore this, we focus on a two-dimensional system consisting of two interacting harmonic oscillators, described by the classical Hamiltonian

$$V(x, y) = \frac{1}{2}(\omega_x^2 x^2 + \omega_y^2 y^2) + \epsilon x y^2.$$

We consider both classical and Bohmian trajectories for increasing values of the coupling parameter ϵ, which introduces a nonlinear interaction between the oscillators.

We first construct a quantum state composed of two coherent states—one in the x direction and one in the y direction—which, in the absence of interaction ($\epsilon = 0$), leads to regular Lissajous-type trajectories. We then study a second quantum state that exhibits a mixed structure, containing both ordered and chaotic regions even when $\epsilon = 0$.

By systematically increasing ϵ, we observe how chaos emerges in both the classical and Bohmian descriptions. In particular, we find that while classical chaos appears as expected from the underlying nonlinear dynamics, Bohmian trajectories also exhibit chaotic behavior over time. In fact, practically all Bohmian trajectories tend to become chaotic in the long run, but the onset of chaos occurs at times that strongly depend on the strength of the interaction between the oscillators.

Our results highlight both qualitative agreements and intriguing differences between classical and quantum dynamics, shedding light on how quantum chaos can emerge in the Bohmian picture.

References

1. A.C. Tzemos and G. Contopoulos, *A comparison between classical and Bohmian quantum chaos*, *Chaos, Solitons and Fractals* **188** (2024) 115524.

2. G. Contopoulos and A.C. Tzemos, *Classical and Bohmian Trajectories in Integrable and Nonintegrable Systems*, Particles **7** (2024) 1062.