A Unified Transform Approach to the Heat Equation on the Half-Line with General Boundary Conditions

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Abstract

This talk presents a detailed application of the *Unified Transform Method* (UTM), also known as the *Fokas Method*, to solve the heat equation posed on the positive half-line under general boundary conditions. We consider the classical PDE:

$$u_t = u_{xx}, \quad 0 < x < \infty, \quad t > 0$$

subject to an initial condition $u(x, 0) = u_0(x)$, and mixed boundary data $u(0, t) = g_0(t)$, $u_x(0, t) = g_1(t)$. Our approach transforms the PDE into a spectral ODE via a Fourier transform on the half-line, followed by the derivation of a global relation linking all known and unknown data.

A key part of the analysis involves the careful study of analyticity and decay properties of the spectral integrand, enabling the deformation of contours into regions where unknown boundary data can be eliminated. This technique results in an explicit, closed-form solution that relies only on known initial and boundary conditions.

We rigorously justify each step, including the use of Jordan's Lemma and asymptotic estimates, and provide detailed visualizations of the complex plane to aid in understanding the method's analytic structure. This framework exemplifies the power and generality of UTM, particularly in contexts where classical methods fail or become unwieldy.

This presentation will be of interest to attendees working in applied mathematics, symbolic computation, PDEs, and mathematical physics.