Mathematical Experiments for Mathematics Majors

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Abstract

At Simon Fraser University I teach a second year course entitled "Computing with Calculus". The course is a required course for all mathematics majors and applied mathematics majors. The prerequisites are an integral calculus course and a first programming course. The course covers one variable calculus, a little bit of multivariate calculus (partial derivatives) and some modelling with first order systems of differential equations. Students attend one lecture hour and one lab hour per week for 12 weeks.

The main goal of the course is to get mathematics majors to use a mathematical software package (I use Maple) to perform a variety of calculations for calculus. Obviously, we want the students to be able to calculate indefinite integrals and definite integrals, solve (systems) of algebraic equations, and solve differential equations. We want them to be able to do these calculations both exactly, numerically, and also visually so that they can see that they are doing.

A second goal is to teach the students how to do a "mathematical experiment". The experiment may be to disprove a conjecture, check a formula, find an optimum solution, or generate an animation of a mathematical object. Doing mathematical experiments usually requires programming, hence the programming prerequisite. Indeed the course provides students a first opportunity to practice their programming skills on mathematical problems instead of more computing problems.

In the talk I will briefly share six mathematical experiments that I've found to be interesting and instructive for students. They are (1) prime number races (See [1]), (2) testing pairs of functions for orthogonality, (3) comparing two numerical methods for measuring river flow (discharge) (See [3]), (4) visualizing eigenvalues and eigenvectors (See [2]), (5) generating random walks in the plane, and (6) simulating a mortgage (a discrete process) in order to compare it with a solution obtained by solving a differential equation.

References

[1] Andrew Granville and Greg Martin. Prime Number Races. *The American Mathematical Monthly* **113**(1):1–33, 2018.

[2] Michael Monagan. 2D and 3D Graphical Routines for Teaching Linear Algebra, *Proceedings of the 2002 Maple Summer Workshop*, Waterloo Maple Inc., 2002.

[3] Michael Monagan. Measuring River Flow. *The Maple Reporter*, Issue 12, Maplesoft, 2013.