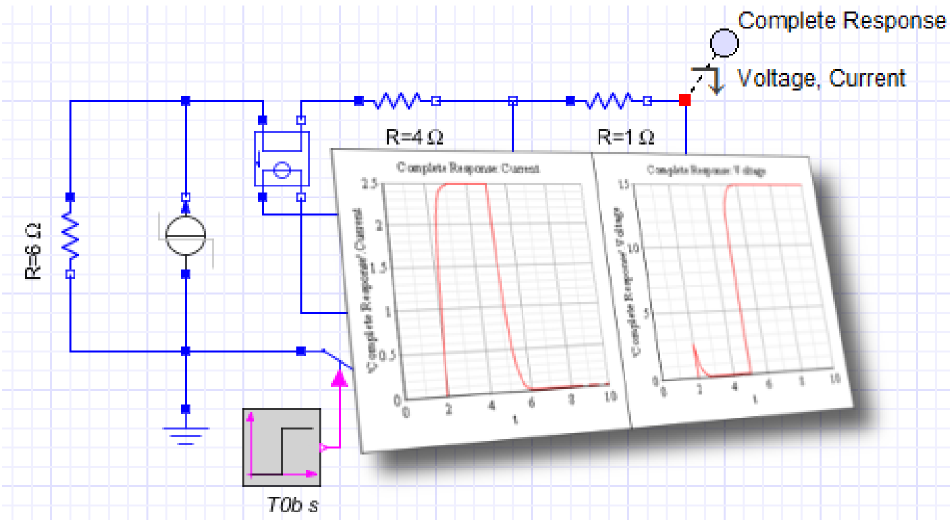
First Order Transient Response



When non-linear elements such as inductors and capacitors are introduced into a circuit, the behaviour is not instantaneous as it would be with resistors. A change of state will disrupt the circuit and the non-linear elements require time to respond to the change. Some responses can cause jumps in the voltage and current which may be damaging to the circuit. Accounting for the transient response with circuit design can prevent circuits from acting in an undesirable fashion.

This section introduces the transient response of first order circuits. It explores the complete response of inductors and capacitors to a state change, including the forced and natural response, and briefly describes a method to solve separable differential equations. The circuits are exposed to constant and exponential voltage or current sources.

First Order Constant Input Circuits

In the case of inductors and capacitors, a circuit can be modeled with differential equations. The order of the differential equations will be equal to the number of capacitors plus the number of inductors. Therefore, we consider a **first order circuit** to be one containing only one inductor or capacitor.

To understand the response of a circuit, we can simplify all elements down to their Norton or Thévenin equivalent circuit for a simpler calculation. If the circuit contains a capacitor, we find the Thévenin equivalent circuit, conversely we find the Norton equivalent if there is an inductor present. If multiple capacitors or inductors are present and these can be combined into an equivalent inductor/capacitor, then we can analyse that circuit as well.

Steady State Response

Consider the circuit in figure 1, shown below.

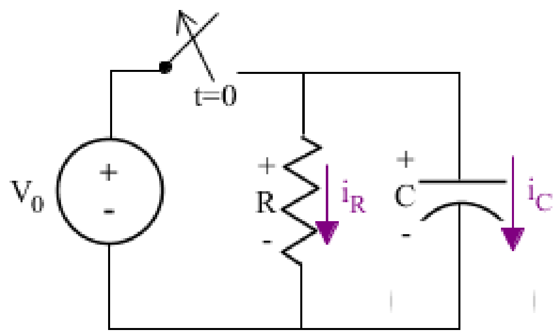


Figure 1: RC circuit

Before t=0, the circuit is at a steady state. A voltage is applied from the voltage source and the circuit is at a steady state. The response or output of the circuit is the voltage across the capacitor. We know that before the switch is opened, the response of the circuit will be a constant V_0. The current will be zero because the voltage is not changing (current through a capacitor is dependant on the derivative of the voltage).

A long time after the switch is opened and the capacitor has discharged, the system will again reach a steady state. The voltage remains constant at zero, and the current is also zero because of the constant voltage across the capacitor. However, immediately after the switch is opened, the circuit enters the **transient** state because it has been disturbed. It takes time to return to a steady state. The **complete response** is both the transient response and the steady state response.

Complete Response = Transient Response + Steady-State Response

Sinusoidal steady states require that the response has the same frequency of the input and is also sinusoidal. Figure 2 demonstrates a sinusoidal circuit entering the transient state at t=0 then reaching steady state after about 7 seconds.

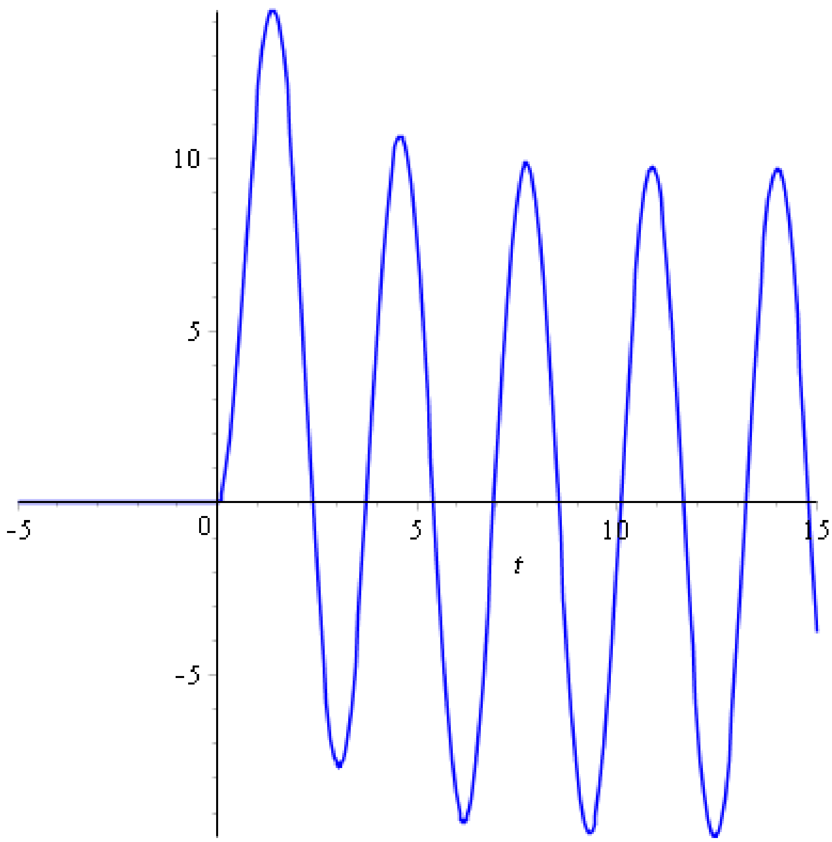


Figure 2: Complete response of an AC circuit

In some contexts, the term *transient* response may refer to the complete response, or the transient response as discussed here. Be careful when using this term.

Natural and Forced Response

The complete response of a circuit can be represented as the sum of the **natural response** and the **forced response**. In a first order circuit, the natural response will be the general solution to the differential equation when the input to the circuit is set to 0.

$$\text{natural response} = A \cdot e^{-\frac{\left(t - t_0\right)}{\tau}}$$

... Eq. (1)

Here, t_0 is the time the change started, tau τ is the **time constant** which determines how quickly the voltage approaches its final value, and A is a constant which affects the amplification of the natural response.

The form of the forced response depends on the input of the circuit. There are 3 cases to consider: the input is a constant, an exponential or a sinusoid. In each, the forced response will have the same form as the input, for example if the input is a sinusoid, the forced response will be a sinusoid with the same frequency. If the input is a constant or exponential, the forced response will also be of that form. The forced response is the steady state response and the natural response is the transient response.

To find the complete response of a circuit,

1. Find the initial conditions by examining the steady state before the disturbance at t_0.
2. Calculate the forced response after the disturbance.
3. Add the natural response of the disturbance to the forced response to obtain the complete response.

There are four cases to consider for first order circuits: A capacitor connected to a Thévenin or Norton circuit, and an inductor connected to a Thevenin or Norton equivalent circuit.

Capacitor and Thévenin Equivalent Circuit

A circuit containing one capacitor has been reduced down to its Thévenin equivalent where the load is the capacitor. We will find the voltage and current across the capacitor.

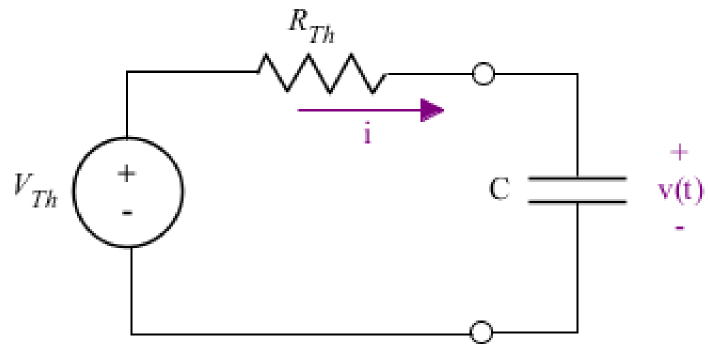


Figure 3: Capacitor and Thevenin circuit

Using a loop, the sum of the voltage will be zero.

$$V_{Th} = R_{Th} \cdot i(t) + v(t)$$

... Eq. (2)

Substitute in the capacitor current.

$$V_{Th} = R_{Th} \cdot \left(C \cdot \frac{\mathrm{d}\, v(t)}{\mathrm{d}t} \right) + v(t)$$

... Eq. (3)

which simplifies into the differential equation,

$$\frac{\mathrm{d}}{\mathrm{d}t} v(t) + \frac{v(t)}{R_{Th} \cdot C} = \frac{V_{Th}}{R_{Th} \cdot C}$$

... Eq. (4)

Move the second term to the right hand side and then divide by the numerator.

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} \cdot \frac{1}{v(t) - V_{Th}} \, \mathrm{d}t = - \frac{1}{R_{Th} \cdot C} \mathrm{d}t$$

... Eq. (5)

The indefinite integral resolves to the following form.

$$\ln\left(v(t) - V_{Th} \right) = - \frac{t}{R_{Th} \cdot C} + \mathrm{D}$$

... Eq. (6)

D is a constant of integration. Removing the natural log and solving for v(t) shows

$$v(t) = V_{Th} + e^{D} \cdot e^{-\left(\frac{t}{R_{Th} \cdot C}\right)}$$

... Eq. (7)

The constant e^D, represented by A, can be found at time t = 0.

$$e^{D} = A = v(0) - V_{Th}$$

... Eq. (8)

We can also solve for the final steady state.

$$v(\infty) = \lim_{t \to \infty} v(t) = V_{Th}$$

... Eq. (9)

Substitute eq. (9) and (8) into eq. (7).

$$v(t) = v(\infty) + (v(0) - v(\infty)) \cdot e^{-\frac{t}{R_{Th} \cdot C}}$$

... Eq. (10)

Set the time constant from the product in the exponential term.

$$\tau = R_{Th} \cdot C$$

... Eq. (11)

Therefore, the final form of the complete reponse is

$$v(t) = v(\infty) + (v(0) - v(\infty)) \cdot e^{-\frac{t}{\tau}}$$

... Eq. (12)

Notice the form of the solution: the forced response (the system at its final steady state, eq. (9)) plus the natural response.

Capacitor and Norton Equivalent Circuit

Figure 4 displays a capacitor connected to a Norton equivalent circuit.

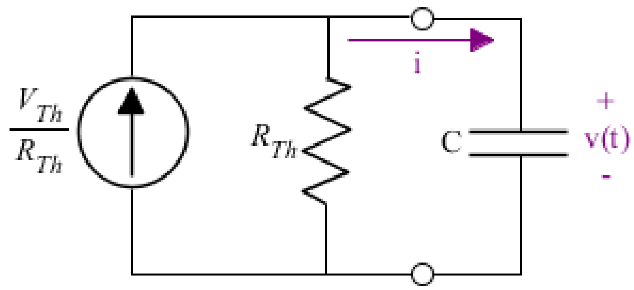


Figure 4: Capacitor with a Norton equivalent circuit

Nodal analysis of the top node reveals the following equation.

$$i(t) + \frac{v(t)}{R_{Th}} = \frac{V_{Th}}{R_{Th}}$$

... Eq. (13)

Substitute the capacitor current.

$$C \cdot \frac{\mathrm{d}\, v(t)}{\mathrm{d}t} = \frac{V_{Th} - v(t)}{R_{Th}}$$

... Eq. (14)

Rearrange into the following form.

$$\frac{\mathrm{d}v(t)}{\mathrm{d}t} \cdot \frac{1}{v(t) - V_{Th}} \mathrm{d}t = -\frac{1}{R_{Th} \cdot C} \mathrm{d}t$$

...r Eq. (15)

This is in the same form as eq. (5). The proof will follow the same steps from eq. (6) to eq. (10), once again resolving to the following form.

$$\tau = R_{Th} \cdot C$$

... Eq. (16)

$$v(t) = v(\infty) + (v(0) - v(\infty)) \cdot \mathrm{e}^{-\frac{t}{\tau}}$$

... Eq. (17)

Inductor and Thévenin Equivalent Circuit

Below is an inductor connected to a circuit which has been reduced to its Thévenin equivalent.

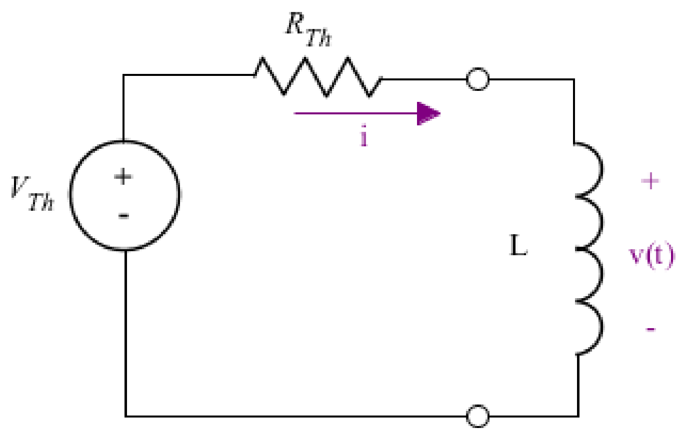


Figure 5: Inductor and Thévenin equivalent circuit

Apply KVL to the loop of this circuit.

$$V_{Th} = R_{Th} \cdot i(t) + v(t)$$

... Eq. (18)

The voltage across an inductor is given by

$$v(t) = L \cdot \frac{d\,i(t)}{dt}$$

... Eq. (19)

Use this in eq. (18).

$$V_{Th} = R_{Th} \cdot i(t) + L \cdot \frac{d\,i(t)}{dt}$$

... Eq. (20)

Rearrange the equation into a form that is easier to integrate.

$$\frac{d\,i(t)}{dt} := \frac{R_{Th}}{L} \cdot \left(\frac{V_{Th}}{R_{Th}} - i(t) \right)$$

... Eq. (21)

Divide by the term in brackets, and integrate.

$$\int \frac{\mathrm{d}\, i(t)}{\mathrm{d}t} \cdot \frac{1}{i(t) - V_{Th} / R_{Th}}\, \mathrm{d}t = -\frac{R_{Th}}{L} \int \mathrm{d}t$$

... Eq. (22)

The integral becomes,

$$\ln\left(i(t) - \frac{V_{Th}}{R_{Th}} \right) = -\frac{t}{L / R_{Th}} + \mathrm{D}$$

... Eq. (23)

Remove the natural log and solve for the inductor current.

$$i(t) = \frac{V_{Th}}{R_{Th}} + \mathrm{e}^{\mathrm{D}} \cdot \mathrm{e}^{-\frac{t}{L / R_{Th}}}$$

... Eq. (24)

At time t = 0, the constant e^{D} = A is revealed.

$$i(0) - \frac{V_{Th}}{R_{Th}} = A = \mathrm{e}^{\mathrm{D}}$$

... Eq. (25)

As the time goes to infinity, the steady state or forced response is found.

$$i(\infty) = \lim_{t \to \infty} i(t) = \frac{V_{Th}}{R_{Th}}$$

... Eq. (26)

The time constant tau τ is,

$$\tau = \frac{L}{R_{Th}}$$

... Eq. (27)

Therefore the complete response of the current through an inductor connected to a thevenin equivalent circuit is

$$i(t) = i(\infty) + (i(0) - i(\infty)) \cdot e^{-\frac{t}{\tau}}$$

... Eq. (28)

Notice the similarities of this form to that of the capacitors?

Inductor and Norton Equivalent Circuit

Consider the circuit shown in the following figure.

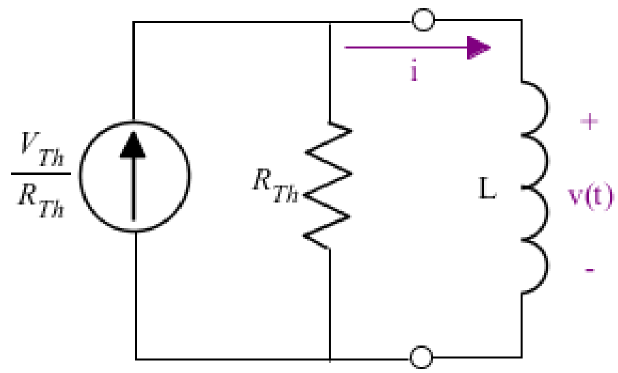


Figure 6: Inductor with Norton equivalent circuit

Nodal analysis of the top node resolves to the following equation.

$$\frac{v(t)}{R_{Th}} + i(t) = \frac{V_{Th}}{R_{Th}}$$

... Eq. (29)

Use the inductor voltage from eq. (19).

$$\frac{L}{R_{Th}} \cdot \frac{d\,i(t)}{dt} + i(t) = \frac{V_{Th}}{R_{Th}}$$

... Eq. (30)

Remove the constants from the derivative.

$$\frac{d\,i(t)}{dt} + \frac{R_{Th}}{L} \cdot i(t) = \frac{V_{Th}}{L}$$

... Eq. (31)

Separating the constants from the current gets this into a form that is easier to integrate.

$$\frac{\mathrm{d}\,i(t)}{\mathrm{d}t} = \frac{R_{Th}}{L} \cdot \left(\frac{V_{Th}}{R_{Th}} - i(t) \right)$$

... Eq. (32)

$$\int \frac{\mathrm{d}\,i(t)}{\mathrm{d}t} \cdot \frac{1}{\left(i(t) - V_{Th} \middle/ R_{Th} \right)}\,\mathrm{d}t = -\frac{R_{Th}}{L} \cdot \int \mathrm{d}t$$

... Eq. (33)

This follows the same proof as eq. (22) to (26). The time constant is therefore,

$$\tau = \frac{L}{R_{Th}}$$

... Eq. (34)

The complete response is

$$i(t) = i(\infty) + (i(0) - i(\infty)) \cdot \mathrm{e}^{-\frac{t}{\tau}}$$

... Eq. (35)

Example: Complete Response with Constant Input

Let's find the differential equation for a circuit with a constant input after time t_0. Consider the circuit with one capacitor and no inductors in figure 1, shown again here.

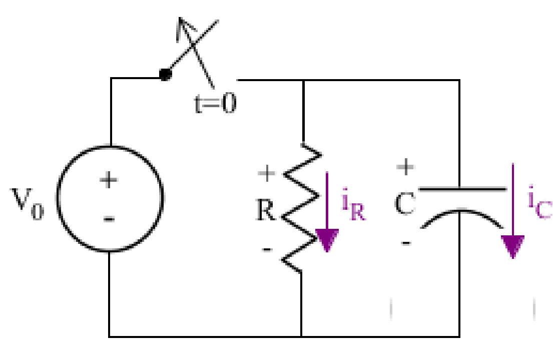


Figure 1: RC circuit with constant input

The first step is to find the initial condition for the voltage at t_0=0. As the circuit is in series and the capacitor will act as an open connection at a steady state, the voltage will be V_0 at t=0.

Once the switch is open and the capacitor discharges (t = ∞), reaching a steady state once again, there will be no potential across the element. The forced response = 0.

When t ≥ 0, the current through the circuit will be

$$i_C(t) + i_R(t) = 0$$

The current through a capacitor is dependant on the rate of change of the voltage, and the resistor current can be found with ohm's law.

$$C \cdot \frac{\mathrm{d}\, v(t)}{\mathrm{d}t} + \frac{v(t)}{R} = 0$$

Rearrange this differential equation into a form that is easier to solve.

$$\frac{1}{v(t)} \cdot \frac{\mathrm{d}\, v(t)}{\mathrm{d}t} \cdot \mathrm{d}t = -\frac{1}{R \cdot C} \cdot \mathrm{d}t$$

Take the definite integral from t0 = 0 to t for each side respectively. Use substitution on the left hand side.

$$\int_{V_0}^{v(t)} \frac{1}{u}\, \mathrm{d}u = -\frac{1}{R \cdot C} \cdot \int_0^t \mathrm{d}\tau$$

Evaluating the integral and solving for the voltage response reveals

$$v(t) = V_0 \cdot \mathrm{e}^{-\frac{t}{RC}} \qquad \text{for} \qquad t \geq 0$$

Therefore, the complete response will be the sum of the natural response and the forced response ($v(\infty) = 0$).

$$v(t) = \begin{cases} V_0 & t \le 0 \\ V_0 \cdot e^{-\frac{t}{RC}} + 0 & t \ge 0 \end{cases}$$

The current across the capacitor will be,

$$i(t) = -\frac{v(t)}{R} = -\frac{V_0}{R} \cdot e^{-\frac{t}{RC}}$$

Complete Response

The four cases demonstrated above all resolve to the same solution. A general form of the complete response should be found.

Proof of the Complete Response

To start, let $x(t)$ represent the parameter of interest, which was voltage $v(t)$ with capacitors and current $i(t)$ with inductors in the previous examples above. The differential equations look similar, so starting from the differential equation from eq. (4),

$$\frac{d}{dt} v(t) + \frac{v(t)}{R_{Th} \cdot C} = \frac{V_{Th}}{R_{Th} \cdot C}$$

... Eq. (4)

Recall the time constant tau τ was the product $R_{Th} \cdot C$. Substitute this in as well as $v(t) = x(t)$ and a constant K, which represents the constant on the right-hand side of the differential equation.

$$\frac{d\,x(t)}{dt} + \frac{x(t)}{\tau} = K$$

... Eq. (36)

Each differential equation can be written in this form. This allows a fast way to obtain the time constant. Let's proceed to solve it.

$$\frac{d\,x(t)}{dt} = \frac{K\cdot\tau - x(t)}{\tau}$$

... Eq. (37)

Factor out -1 and divide the numerator on the right hand side.

$$\frac{d\,x(t)}{dt}\cdot\frac{1}{\left(x(t) - K\cdot\tau\right)} = -\frac{1}{\tau}$$

... Eq. (38)

Integrate the differential equation.

$$\int \frac{d\,x(t)}{dt}\cdot\frac{1}{x(t) - K\cdot\tau}\,dt = -\frac{1}{\tau}\cdot\int dt$$

... Eq. (39)

The integral becomes

$$\ln\left(x(t) - K\cdot\tau\right) = -\frac{t}{\tau} + D$$

... Eq. (40)

Remove the natural log and let A = e^{D}.

$$x(t) = K\cdot\tau + A\cdot e^{-\frac{t}{\tau}}$$

... Eq. (41)

This maps a solution from the differential equation to the complete response. The constants are also represented by the steady state response.

$$x(0) = K\cdot\tau$$

... Eq. (42)

$$x(\infty) - x(0) = A$$

... Eq. (43)

In general, first order RL and RC circuits have a response following the form,

$$x(t) = B + (A - B) \cdot e^{-t/\tau}$$

... eq. (44)

with the time constant tau τ, the initial value A and final value B.

Adjust the sliders below and observe the effect on the complete response of the circuit. The slider for the parameter *A* controls the initial voltage or current at t ≤ 0. *B* is the forced or steady-state response when t ≥ 0. Finally, tau τ is the time constant.

A = 0

B = 20

Time constant τ =
11

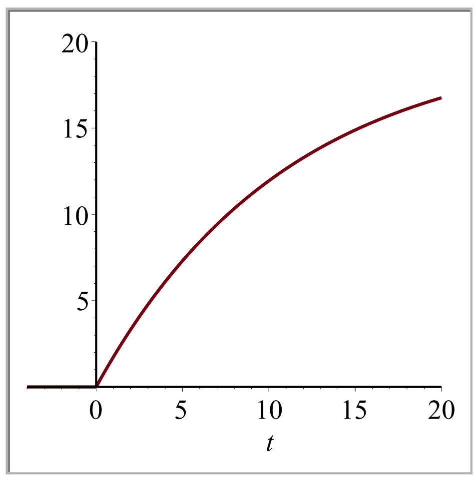


Table 1: Complete response interactive chart

Sequential Switching

Some circuits have multiple stages at which they change states. **Sequential switching** occurs in a circuit which changes states two or more times at different moments. Solving these circuits require the same methods previously described. The consecutive switches have initial conditions which can be found using the response of the first switch at that time instance.

Example: Sequential switching

Consider the circuit shown below.

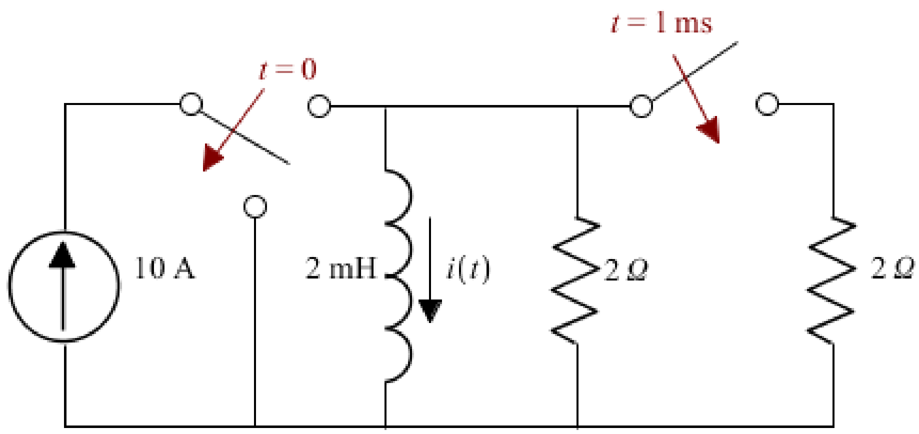


Figure 7: Inductor circuit with two state changes

There are two switches which execute at different instances. To begin, let's find the initial conditions prior to both switches.

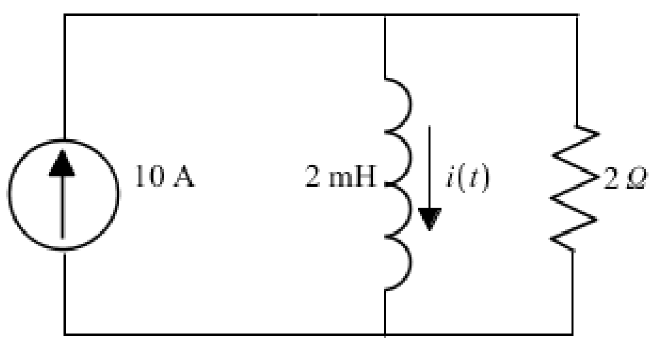


Figure 8: Circuit before t=0

The inductor at a steady state will act as a short circuit, therefore it will have ten amps flowing through it up to immediately before the switch. After the switch, the inductor behaves as such and the circuit looks like this:

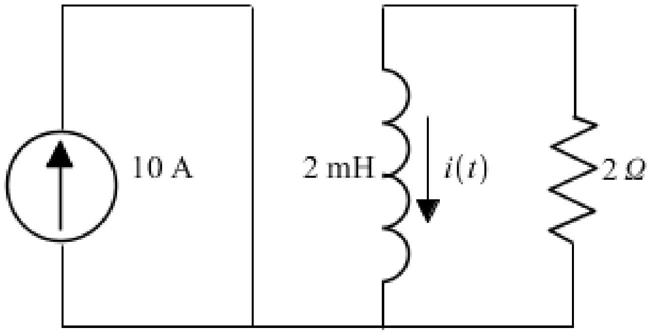


Figure 9: Circuit after t=0 and before second switch

The current source no longer supplies any power, so the inductor will discharge. We know the initial state, and the final state has no current because the inductor will discharge. Use eq. (35) to find the natural response.

$$i(t) = i(0) \cdot e^{-\frac{t}{\tau}}$$

The time constant is,

$$\tau = \frac{L}{R} = \frac{2 \cdot 10^{-3}}{2} = 1 \text{ ms}$$

Assume t is measured in milliseconds. Therefore before the second switch, the circuit has the response,

$$i(t) = 10 \cdot e^{-t}$$

Using this response, the circuit right before the second switch at t=1 ms, the current will be

$$i(1) = 10 \cdot e^{-1} = i(1) = 3.6788$$

3.68 amps. The circuit changes state at t=1 ms.

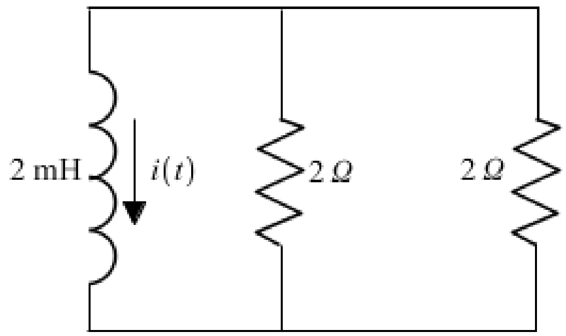


Figure 10: Circuit after t=1 ms.

The second switch adds an additional resistor. The equivalent resistance is,

$$R = \left(\frac{1}{2} + \frac{1}{2}\right)^{-1} = 1$$

An equivalent resistance of 1 ohm. We know the initial condition before the switch. Again, in its final steady state there is no current. The time current will change.

$$\tau = \frac{L}{R} = \frac{2 \cdot 10^{-3}}{1} = 2 \text{ ms}$$

The complete response of the circuit therefore becomes,

$$i(t) = \begin{cases} 10 & t < 0 \\ 10 \cdot e^{-t} & 0 < t < 1\ ms \\ 3.68 \cdot e^{-\frac{(t-1)}{2}} & 1\ ms < t \end{cases}$$

Exponential Sources

The previous examples described the response to a constant source. What will be the response if a capacitor or inductor is connected to an exponential source?

The general differential equation describing the response of a circuit is

$$\frac{\mathrm{d}\,x(t)}{\mathrm{d}t} + a\cdot x(t) = y(t)$$

... Eq. (45)

Where a is 1/τ. Prior to now, y(t) was considered a constant, K. Now that the differential equation is not separable, we must use a different method. Consider the derivate which expands with product rule shown below.

$$\frac{\mathrm{d}}{\mathrm{d}t}\left(x\cdot e^{at}\right) = \frac{\mathrm{d}x}{\mathrm{d}t}\cdot e^{at} + a\cdot x\cdot e^{at} = \left(\frac{\mathrm{d}x}{\mathrm{d}t} + a\cdot x\right)\cdot e^{at}$$

... Eq. (46)

If we multiply eq. (45) by the exponent e^{at} and integrate, the left hand side will resolve to $x\cdot e^{at}$.

$$\int \frac{\mathrm{d}}{\mathrm{d}t}\left(x\cdot e^{at}\right)\,\mathrm{d}t = \int y\cdot e^{at}\,\mathrm{d}t$$

... Eq. (47)

The derivative and integral cancel each other out on the left hand side. Remove the exponent from the left side, and add a constant of integration, K.

$$x = e^{-at}\cdot\int y\cdot e^{at}\,\mathrm{d}t + K\cdot e^{-at}$$

... Eq. (48)

Notice that the natural response is still of the form $K\cdot e^{at}$. Assume that if y(t) is exponential, it is of the form e^{bt}. We can now evaluate the integral.

$$x = e^{-at}\cdot\int e^{(a+b)\cdot t}\,\mathrm{d}t + K\cdot e^{-at}$$

... Eq. (49)

The integral evaluates to

$$x = e^{-at}\cdot\frac{1}{a+b}\cdot e^{(a+b)\cdot t} + K\cdot e^{-ay}$$

... Eq. (50)

Simplify the exponential terms to obtain the general form.

$$x(t) = \frac{e^{bt}}{a+b} + K \cdot e^{-at}$$

... Eq. (51)

We must assume the sum of a and b is not equal to zero.

Example: Exponential Source

The current source in this circuit turns on at t=0 and generates a current at an exponential rate.

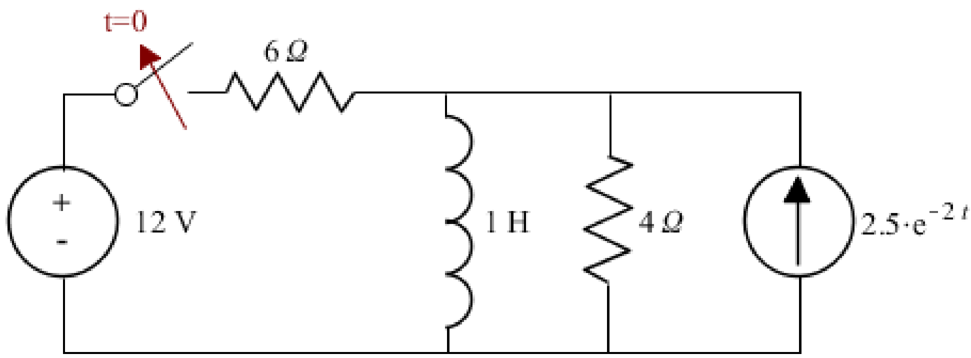


Figure 11: LR circuit with an exponential source

The first step is to obtain the initial values. The circuit will be in a steady state prior to t=0, and the exponential current source will be off and act as an open circuit.

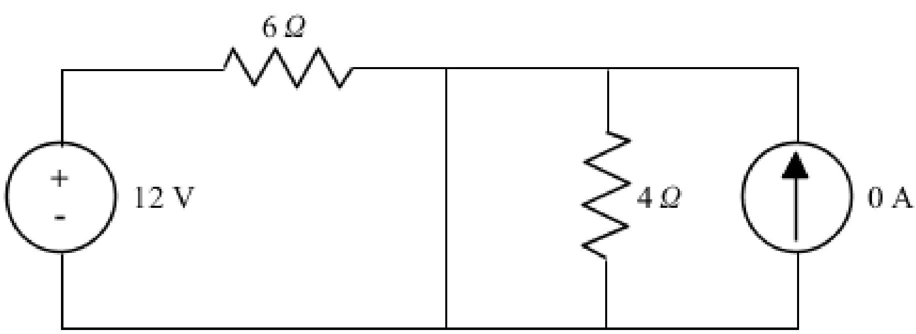


Figure 12: Circuit before t=0

The 4 ohm resistor can be omitted due to the short circuit at the inductor. The current across the inductor is found with ohm's law.

$$i = \frac{V}{R} = \frac{12}{6} = 2\,A$$

We now have the initial conditions. The circuit after the switch opens becomes

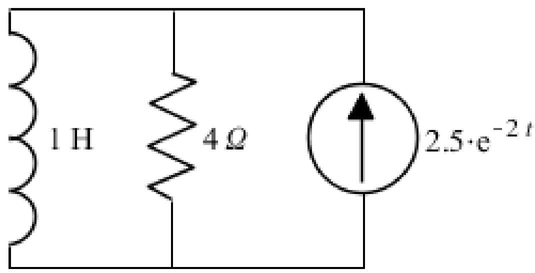


Figure 13: Circuit after t=0

The natural response is easily found with the circuit in this form, using eq. (34) and (35).

$$\tau = \frac{L}{R} = \frac{1}{4}$$

$$i_n = A \cdot e^{-\frac{t}{1/4}} = A \cdot e^{-4t}$$

We can expect the forcing function to be of the same form as the current source after the switch opens.

$$i_f = B \cdot e^{-2t}$$

Using KCL at the top node, find the differential equation.

$$i(t) + \frac{L}{R} \cdot \frac{d\,i(t)}{dt} = 2.5 \cdot e^{-2t}$$

Substitute the assumed forced current.

$$-2 \cdot B \cdot e^{-2t} + 4 \cdot B \cdot e^{-2t} = 10 \cdot e^{-2t}$$

Remove the exponential terms. The equation resolve to find B = 5. The complete response for t > 0 is

$$i(t) = i_f + i_n = 5 \cdot e^{-2t} + A \cdot e^{-4t}$$

Using the initial condition $i(t=0) = 2$, the coefficient A is found to be A = -3. Therefore, the complete response is

$$i = \begin{cases} 2 & t < 0 \\ 5 \cdot e^{-2 \cdot t} - 3 \cdot e^{-4 \cdot t} & t \geq 0 \end{cases}$$

This response is displayed below.

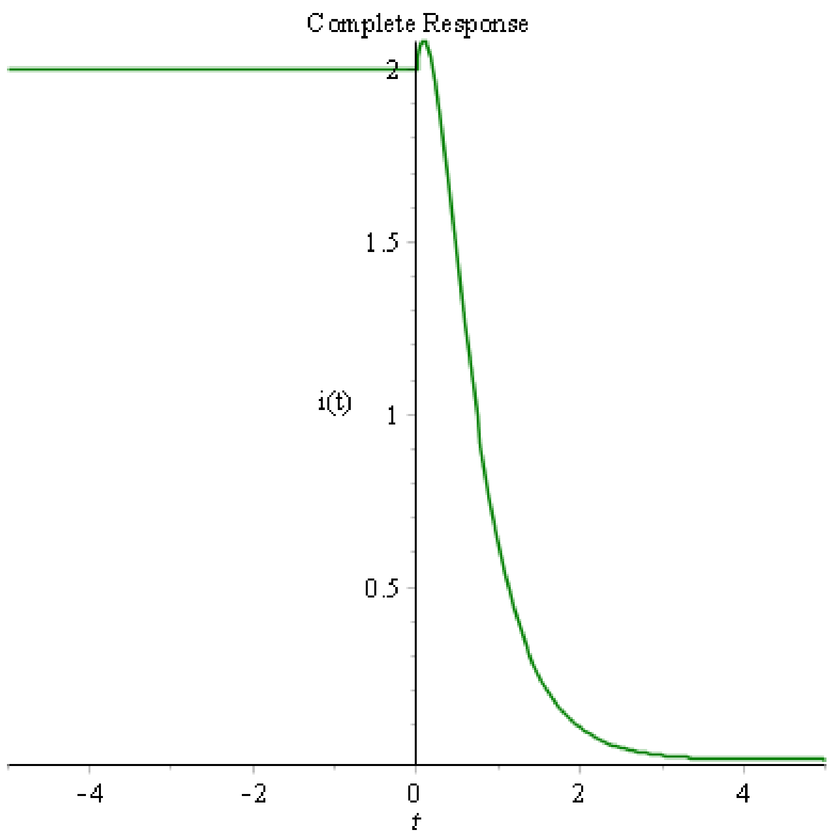


Figure 14: Complete response of LR circuit with exponential source.

Examples with MapleSim

Example 1: Complete Response with Constant Sources

Problem Statement: Find the complete response in the following circuit.

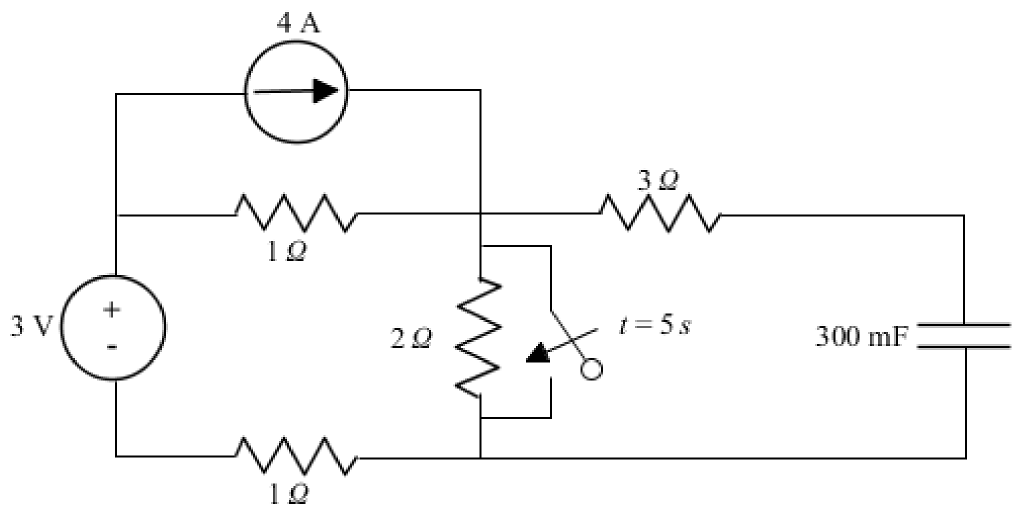


Figure 15: Constant sources with a capacitor

Analytical Solution

restart;

Data:

$Vs := 3:$

$Is := 4:$

$R1 := 1:$

$R2 := 1:$

$R3 := 2:$

$R4 := 3:$

$C := 0.3:$

$t_0 := 5:$

Solution:

First, we must find the initial conditions of the circuit. Calculate the Thevenin equivalent circuit in a steady state before t = 5. Find the equivalent voltage by making an open circuit at the load (at the capacitor).

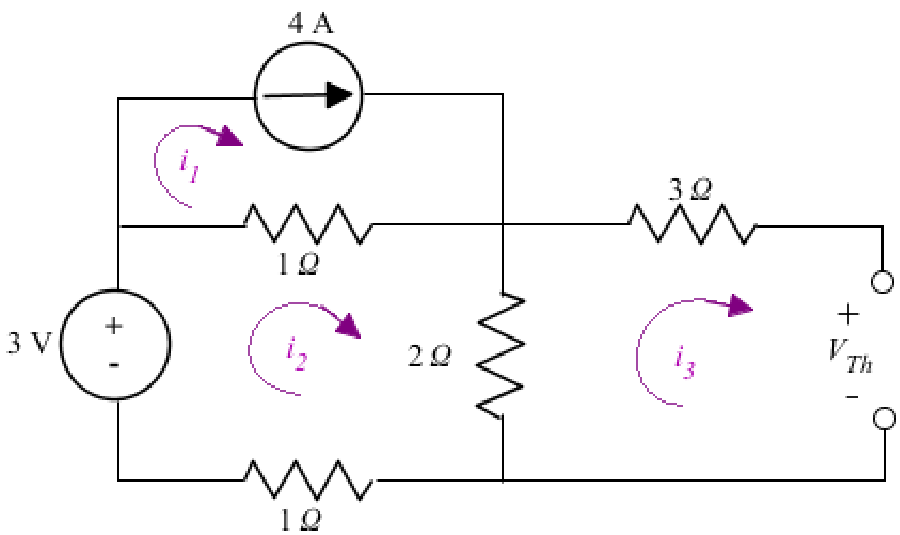


Figure 16: Thevenin open circuit

Current i_1 will be the same as the voltage source, Is. The third loop current i_3 has an open segment and therefore zero current. The final loop finds the current of i_2.

$i_1 := Is :$

$i_3 := 0 :$

$$i_2 \cdot R2 - Vs + (i_2 - i_1) \cdot R1 + (i_2 - i_3) \cdot R3 = 0 = i_2 = \frac{7}{4}$$

$$i_2 := \frac{7}{4} :$$

Because there is no current at i_3, there is no voltage drop at the 3 ohm resistor and therefore the voltage is equal on both sides.

$$V_{Th} := i_2 \cdot R3 = \frac{7}{2}$$

Optional: Find the equivalent resistance by making the current source an open circuit and the voltage source a short circuit.

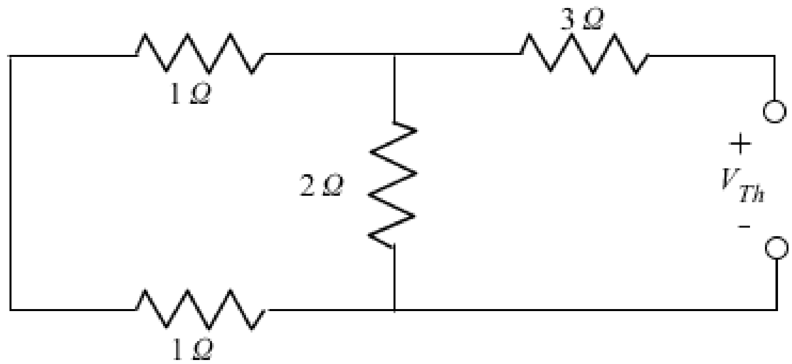


Figure 17: Thevenin resistance

$$R_{Th} := \left(\frac{1}{(R1+R2)} + \frac{1}{R3}\right)^{-1} + R4 = 4$$

Now that we know the initial conditions, find the final steady state circuit. This will be used to find the forced response.

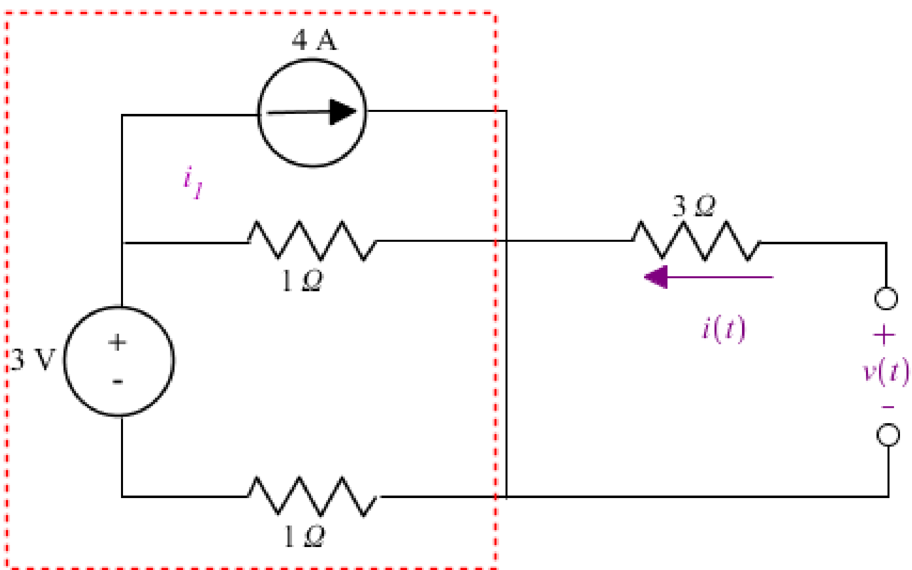


Figure 18: Forced response state

The elements within the red box are omitted due to the short circuit. Therefore by analysis, at a steady state the current and the voltage will be zero.

Now we can find the natural response of the system. Do KVL around the loop after the switch closed.

$$3 \cdot i(t) + v(t) = 0$$

Substitute the current of a capacitor.

$$3 \cdot C \cdot \frac{\mathrm{d}\, v(t)}{\mathrm{d}t} + v(t) = 0$$

Rearranging the differential equations forms the general complete response. Obtain the time constant and K value.

$$\frac{\mathrm{d}\, v(t)}{\mathrm{d}t} + \frac{v(t)}{3 \cdot C} = 0$$

$$\tau = 3 \cdot C = 0.9 :$$

$$K = 0 :$$

Plug these into the compete response from eq. (41).

$$v(t) = 0 + (v(0) - v(\infty)) \cdot \mathrm{e}^{-\frac{t}{0.9}}$$

$$v(t) = 3.5 \cdot \mathrm{e}^{-\frac{t}{0.9}}$$

Of course, this had assumed $t_0 = 0$. Accounting for the time shift, the complete response of the voltage becomes

$$v(t) := \begin{cases} 3.5 & t < t_0 \\ 3.5 \cdot \mathrm{e}^{-\frac{t-5}{0.9}} & t \geq t_0 \end{cases} :$$

The capacitor current is found by deriving the voltage.

$$i(t) = C \cdot \frac{d}{dt}(v(t)) = C \cdot \frac{d}{dt}\left(3.5 \cdot e^{-\frac{t-0.5}{0.9}}\right) = -\left(0.3 \cdot \frac{3.5}{0.9}\right) \cdot e^{-\frac{t-5}{0.9}}$$

Therefore, the current is

$$i(t) := \begin{cases} 0 & t < t_0 \\ -1.1666 \cdot e^{-\frac{t-5}{0.9}} & t \geq t_0 \end{cases} :$$

The figures below display the response of the system.

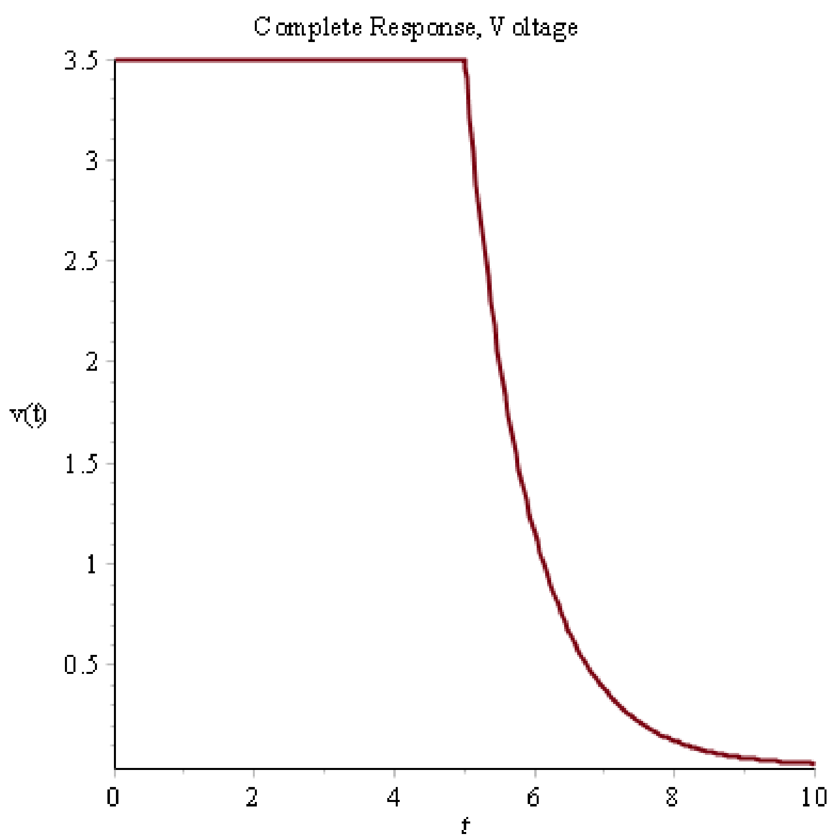


Figure 19: Voltage Response

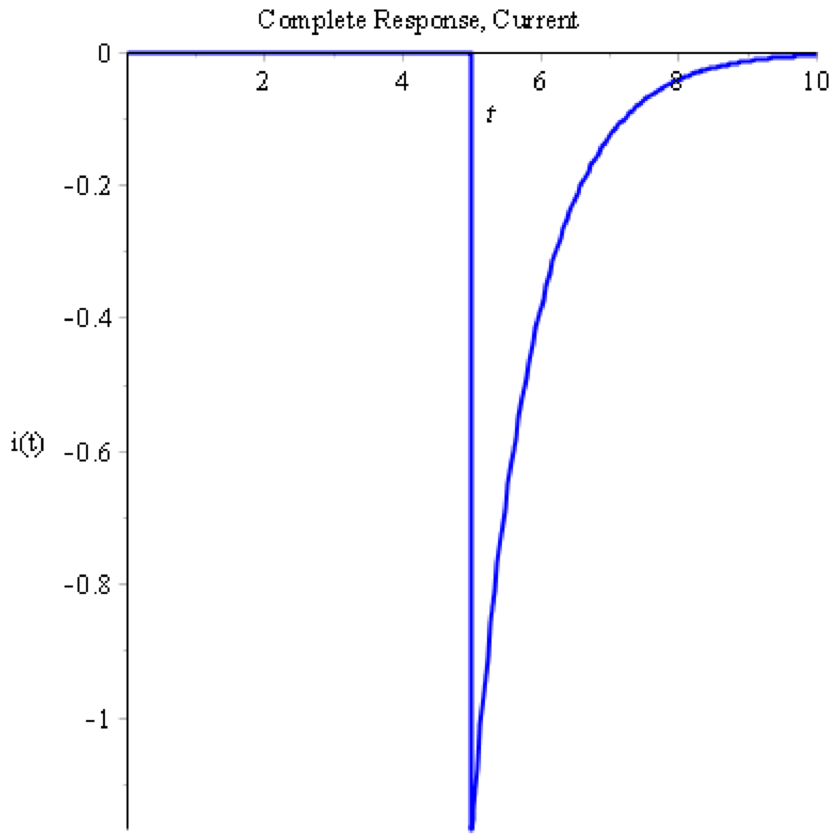
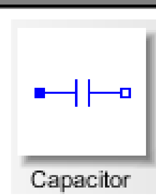


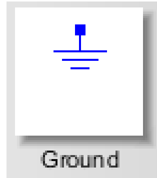
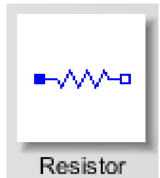
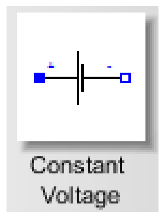
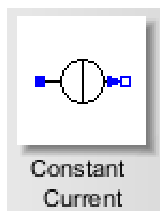
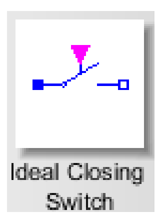
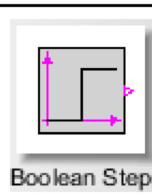
Figure 20: Current Response

MapleSim Solution

Step 1: Insert Components

Drag the following components into a new workspace.

Component	Location
	Electrical > Analog > Common

	Electrical > Analog > Common
 (4 Required)	Electrical > Analog > Common
	Electrical > Analog > Sources > Voltage
	Electrical > Analog > Sources > Current
	Electrical > Analog > Switches
	Signal Blocks > Sources > Boolean

Step 2: Connect the components.

Connect the components as shown in the diagram below.

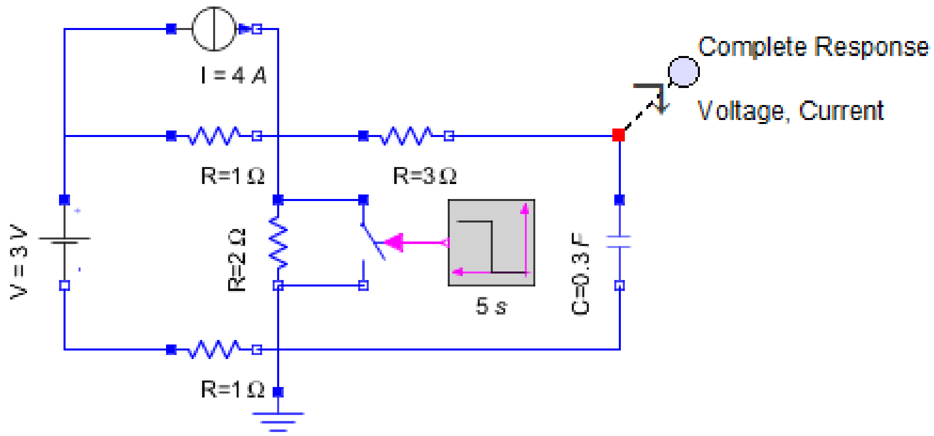


Figure 21: MapleSim Model Diagram

Step 3: Set up parameters

1. Select the **Constant Current** block. On the 'Inspector' tab, set the current I parameter to **4**.
2. Select the **Constant Voltage** block. Set the voltage to **3** volts.
3. For each resistor, set the appropriate value as specified in the model diagram and problem statement. The resistor values should be **1** for the top-left, **1** for the bottom left, **2** for the middle and **3** for the top-right resistor.
4. Select the **Boolean Step** block and set the step start time to **5** seconds.
5. Select the **Capacitor** block and set the capacitance *C* to **0.3** farads.

Step 4: Connect probes

1. Connect a probe  to the line between the resistor and the capacitor. On the 'Inspector' tab, make sure both boxes are **checked** to measure the voltage *v*, and current, *i*.

Step 5: Run Simulation

Run the simulation to observe the complete response of the capacitor.

Results

The following plots are generated upon running the simulation.

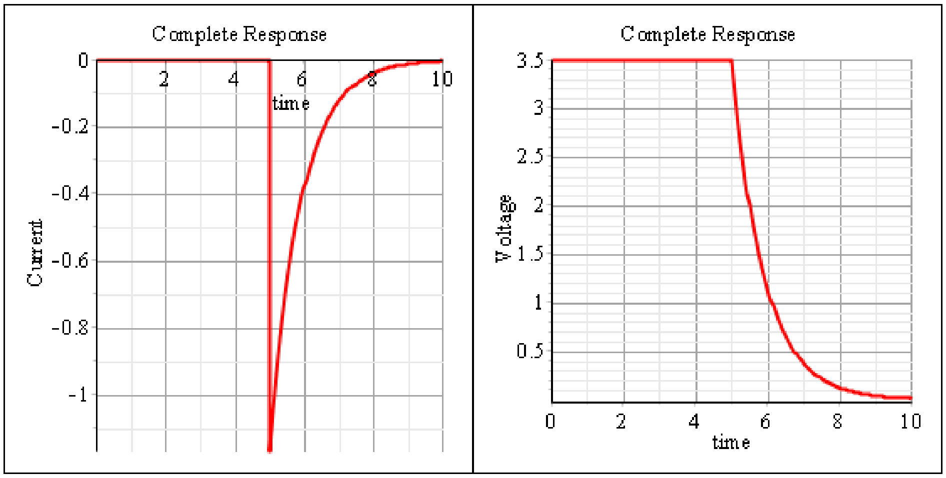


Figure 21: MapleSim simulation results

References:

R. Dorf, J. Svoboda. "Introduction to Electric Circuits", 8th Edition. RRD Jefferson City, 2010, John Wiley and Sons, Inc.

S. Prasad. *First Order Circuits* (version 1.65) [PDF document]. Retrieved from Portland State University website at http://web.cecs.pdx.edu/~prasads/FirstOrderCircuits.pdf