A Comparison of Maple™ and Mathematica®: Solving ODEs

The famous text, *Differentialgleichungen* by Kamke, contains 1429 linear and non-linear ODEs. These equations were used to test the symbolic ODE solving abilities of Maple™ and Mathematica®. The tests looked at their ability to find solutions to the 1345 ODEs from the text known of have a solution, as well as how long it takes them to recognize when they cannot solve the problem.

Maple

Solves 97.5% of solvable ODEs
Completes test in 45 minutes

Mathematica

Solves 80.3% of solvable ODEs
Completes test in 7 hours 8 minutes

Kamke ODEs	Maple					Mathematica				
	Solvable Problems			All		Solvable Problems			All	
	Returned Solution	**Not Solved**	**% Solved**	**No response within time limit (3 min)**	**Total Time Spent**	**Returned Solution**	**Not Solved**	**% Solved**	**No response within time limit (3 min)**	**Total Time Spent**
1st order: 576 solvable: 547	541	6	99%	2	19 min	473	74	86%	65	250 min
2nd order nonlinear: 188 solvable: 162	157	5	97%	2	13 min	114	48	70%	8	63 min
2nd order nonlinear reducible 58	50	8	86%	0	3 min	0	58	0%	1	23 min
2nd order linear 448 solvable: 427	424	3	99%	1	5 min	387	40	91%	8	40 min
3rd order linear: 85 solvable: 80	73	7	91%	1	3.5 min	58	22	72%	4	28 min
4th order linear: 45 solvable: 43	40	3	93%	0	22 sec	33	10	77%	1	8 min
5th order linear: 12	11	1	92%	0	24 sec	9	3	75%	2	13 min
High order nonlinear: 12 solvable: 11	11	0	100%	0	21 sec	6	5	54%	1	2.5 min
High order nonlinear reducible: 5	5	0	100%	0	7 sec	0	5	0%	0	30 sec
Totals for the 1429 equations of which 1345 are solvable	1312	33	**97.5%**	6	**45 min**	1080	265	**80.3%**	90	**7.1 hours**

Details

This comparison was done using Maple 17.02 and Mathematica 9.0.1.

The 1429 Ordinary Differential Equations (ODE) of Kamke's book were tackled with Mathematica's `DSolve` and Maple's `dsolve` commands, with a time limit of 3 minutes per ODE (Maple function `timelimit` and Mathematica function `TimeConstrained`), in a Macbook Pro (2013) 2.7 GHz Intel Core I7, 4 cores, 16 GB of RAM. No optional arguments were allowed when calling `DSolve` and `dsolve`.

The test focused on: time consumed, number of equations solved versus unsolved, and also number of "no response within time limit" equations, as a measure of the ability of the system to understand that a problem is beyond its capabilities.

In this report we say that the ODE solver gave "no response within time limit" for an equation when the solver did not terminate within a three minutes time limit. At that point, the computation was automatically interrupted, with the system moving to tackling the next equation.

By "Not Solved" equations it is meant equations for which we know a solution exists (either because Kamke shows it or because we were able to derive it and test it), and for which the ODE solver terminates before 3 minutes or returns without a solution. Problems for which the returned answer involved DESol in Maple or the equivalent DifferentialRoot in Mathematica are considered unsolved, because the answer does not provide more information than the original problem.

Regarding the unsolvable equations of Kamke's book, naturally, neither Mathematica nor Maple are expected to solve them, but it is expected that these systems recognize their inability to solve these equations within a reasonable period of time.

To compare the time spent in tackling only the portion of the 1429 ODEs for which the solver returns before the time limit, multiply the number of "no response within time limit" equations by 3 minutes and then subtract this number from the total time consumed.

First order ODEs

There are 576 first order ODEs in Kamke's book: 29 are unsolvable and 547 are solvable. The Kamke's numbers for the unsolvable ODEs are 47, 48, 49, 50, 55, 56, 74, 79, 82, 87, 202, 203, 205, 206, 219, 234, 237, 250, 253, 265, 269, 331, 367, 370, 395, 461, 503, 572, 576.

Mathematica performance: 86.5 % in 4 hours and 10 minutes

Mathematica took 4 hours and 10 minutes in tackling the 576 ODEs, solved 473 of the 547 solvable ones and failed to return a solution for 74 solvable equations: 16, 22, 63, 66, 69, 70, 80, 81, 83, 86, 121, 127, 266, 292, 340, 365, 366, 368, 383, 385, 394, 400, 402, 404, 413, 414, 416, 428, 429, 451, 452, 460, 465, 467, 468, 479, 482, 485, 487, 489, 494, 504, 506, 508, 509, 510, 513, 515, 520, 523, 524, 527, 528, 530, 532, 533, 534, 535, 537, 538, 541, 542, 543, 544, 546, 550, 555, 561, 562, 566, 567, 570, 575.
Mathematica does not respond within the time limit for 65 equations: 22, 63, 66, 69, 70, 86, 127, 219, 250, 265, 266, 269, 292, 340, 365, 366, 368, 383, 385, 400, 402, 404, 413, 414, 416, 428, 429, 451, 452, 465, 467, 468, 479, 482, 485, 487, 489, 494, 503, 504, 509, 513, 515, 520, 523, 524, 527, 528, 530, 532, 533, 534, 535, 537, 538, 541, 542, 543, 544, 546, 550, 555, 561, 562, of which 60 equations are solvable.

Maple performance: 99 % in 19 minutes

Maple took 19 minutes in tackling the 576 ODEs, solved 541 of the 547 solvable ones and failed to return a solution for 6 solvable equations: 121, 340, 460, 506, 510, 575. Maple does not respond within the time limit for 2 equations: 331, 340, of which 340 is solvable.

Second order nonlinear ODEs

There are 246 second order nonlinear ODEs in Kamke's book, of which 26 are unsolvable and 220 are solvable, divided into fully solvable (162 of them) and reducible (58 of them).

The Kamke's numbers of the 26 unsolvable ODEs are: 3, 5, 6, 8, 9, 13, 16, 18, 19, 27, 29, 52, 55, 59, 85, 95, 112, 114, 139, 147, 149, 161, 207, 211, 212, 217.

The Kamke numbers of the 58 second order nonlinear ODEs that are only reducible are: 11, 15, 21, 22, 25, 26, 28, 34, 36, 37, 44, 46, 47, 49, 53, 54, 58, 68, 69, 70, 72, 73, 74, 75, 76, 77, 82, 83, 87, 90, 92, 94, 96, 100, 102, 103, 105, 106, 115, 116, 118, 123, 129, 131, 144, 148, 152, 172, 187, 198, 199, 225, 226, 230, 231, 235, 242, 246.

Maple conveys solutions for the reducible equations using 'ODESolStruc', showing the solution for the original problem in terms of the solution of the reduced ODE, as well as the reduced ODE and transformation used to reduce the problem. Mathematica does not convey reductions of order, thus failing with all the 58 equations of 'only reducible to first order' type.

Mathematica performance: solvable: 70.4 %, reducible 0 %, in 1 hour and 16 minutes

Mathematica took 1 hour and 16 minutes in tackling the 246 ODEs, solved 114 of the 162 solvable ones, 0 of the 58 reducible ones, and failed to return a solution for 48 fully solvable equations: 23, 24, 30, 31, 32, 33, 35, 38, 39, 41, 43, 45, 48, 62, 66, 88, 91, 101, 108, 119, 120, 130, 142, 145, 156, 165, 167, 171, 189, 190, 208, 216, 219, 223, 227, 228, 229, 233, 237, 241, 244, 20, 50, 98, 121, 186, 221, 243. Mathematica does not respond within the time limit for 8 equations: 20, 50, 98, 121, 186, 221, 230, 243, of which 7 equations are fully solvable.

Maple performance: solvable: 97 %, reducible: 86 %, in 16 minutes

Maple took 16 minutes in tackling the 246 ODEs, solved 157 of the 162 solvable ones, 50 of the 58 reducible ones, and failed to return a solution for 5 fully solvable equations: 108, 142, 145, 167, 171. Maple does not respond within the time limit for 2 equations: 217, 245, of which 1 equation is fully solvable.

Second order linear ODEs

There are 448 second order linear ODEs in Kamke's book, of which 21 are unsolvable and 427 are solvable. The Kamke's numbers for the unsolvable ODEs are 15, 19, 26, 28, 30, 31, 38, 72, 73, 75, 76, 77, 205, 212, 216, 236, 278, 408, 439, 440, 443.

Mathematica performance: 91 %, in 40 minutes

Mathematica took 40 minutes in tackling the 448 ODEs, solved 387 of the 427 solvable ones and failed to return a solution for 40 solvable equations: 27, 29, 32, 80, 81, 82, 83, 84, 85, 128, 156, 413, 418, 442, 445, 157, 248, 261, 268, 303, 306, 329, 330, 343, 348, 362, 367, 372, 373, 398, 402, 403, 177, 232, 233, 263, 341, 406, 407, 427. Mathematica does not respond within the time limit for 8 equations: 177, 232, 233, 263, 341, 406, 407, 427, all of which are solvable.

Maple performance: 99 %, in 5 minutes

Maple took 5 minutes in tackling the 448 ODEs, solved 424 of the 427 solvable ones and failed to return a solution for 3 solvable equations: 81, 57, 441. Maple does not respond within the time limit for 1 equation: 441.

Third order linear ODEs

There are 85 third order linear ODEs in Kamke's book, of which 5 are unsolvable and 80 are solvable. The Kamke's numbers for the unsolvable ODEs are 9, 10, 11, 12, 28.

Mathematica performance: 72.5 %, in 28 minutes

Mathematica took 28 minutes in tackling the 85 ODEs, solved 58 of the 80 solvable ones and failed to return a solution for 22 solvable equations: 13, 14, 15, 23, 24, 25, 26, 59, 62, 67, 81, 82, 83, 36, 41, 57, 72, 79, 2, 34, 68, 78. Mathematica does not respond within the time limit for 4 equations: 2, 34, 68, 78, all of which are solvable.

Maple performance: 91 %, in 3.5 minutes

Maple took 3.5 minutes in tackling the 85 ODEs, solved 73 of the 80 solvable ones and failed to return a solution for 7 solvable equations: 13, 14, 25, 36, 41, 62, 83. Maple does not respond within the time limit for 1 equation: 67, which is solvable.

Fourth order linear ODEs

There are 45 fourth order linear ODEs in Kamke's book, of which 2 are unsolvable and 43 are solvable. The Kamke's numbers for the unsolvable ODEs are 7, 8.

Mathematica performance: 76.7 %. in 8 minutes

Mathematica took 8 minutes in tackling the 45 ODEs, solved 33 of the 43 solvable ones and failed to return a solution for 10 solvable equations: 9, 10, 11, 14, 40, 41, 42, 43, 39, 19. Mathematica does not respond within the time limit for 1 equation: 19, which is solvable.

Maple performance: 93 %, in 22 seconds

Maple took 22 seconds in tackling the 45 ODEs, solved 40 of the 43 solvable ones and failed to return a solution for 3 solvable equations: 9, 10, 11. Maple responds within the time limit for all equations

Fifth order linear ODEs

There are 12 fifth order linear ODEs in Kamke's book, all of them solvable.

Mathematica performance: 75 %, in 13 minutes

Mathematica took 13 minutes in tackling the 12 ODEs, solved 9 of the 12 solvable equations and failed to return a solution for 3 solvable equations: 8, 5, 12. Mathematica does not respond within the time limit for 2 equations: 5, 12, which are both solvable.

Maple performance: 92 %, in 24 seconds

Maple took 24 seconds in tackling the 12 ODEs, solved 11 of the 12 solvable equations and failed to return a solution for 1 solvable equation: 4. Maple responds within the time limit for all equations.

Higher order nonlinear ODEs

There are 17 higher order nonlinear ODEs in Kamke's book, of which 1 is unsolvable and 16 are solvable, in turn divided into fully solvable (11 of them) and fully reducible (5 of them). The Kamke's number for the unsolvable ODE is 15.

The Kamke numbers of the 5 higher order ODEs that are only reducible are: 2, 3, 4, 5, 14.

Mathematica performance with solvable ODEs: 54.6 %; with reducible ODEs: 0 %, in 3 minutes

Mathematica took 3 minutes in tackling the 17 ODEs, solved 6 of the 11 solvable ones, 0 of the 5 reducible ones, and failed to return a solution for 5 fully solvable equations: 1, 8, 9, 17, 12. Mathematica does not respond within the time limit for 1 equation: 12, which is fully solvable.

Maple performance with solvable ODEs: 100%; with reducible ODEs: 100%, in 28 seconds

Maple took 24 seconds in tackling the 17 ODEs. Maple responds within the time limit for all equations, and solves both the solvable and reducible sets of equations completely.